ACCELERATING REFERENCE FRAMES: INERTIAL "FORCES" AND LOCAL ACCELERATION DUE TO GRAVITY

It is often helpful to analyze a physical situation from a reference frame that moves with (translational) acceleration $\vec{A}$ relative to an inertial frame. In the accelerating frame we can use Newton's Second Law to predict or account for the motion of any object as long as we attribute a fictitious "force" called an *inertial "force,"* $\vec{F}'_{inertial} = -m\vec{A}$, to that object (where *m* is the mass of the object).

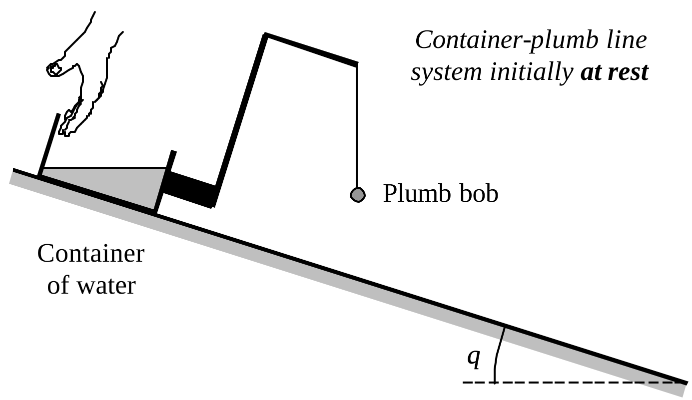
I. Inertial "force"

A. A container of water is at rest on a level table. A stationary plumb bob is suspended nearby. (Treat the water as an incompressible fluid.)

How would you describe the orientation of the plumb line relative to the surface of the water in the container?

B. Suppose that the container of water were now placed on a long, frictionless incline. The support stand for the plumb line is attached to the container so that both may move together. Define frame *O*¢ to be a reference frame that is at rest with respect to the container-plumb line system.

When answering the following questions, consider an instant long after the system has been released, *i.e.*, when the plumb line remains at a *fixed orientation*.



1. Show that the acceleration of the container-plumb line system relative to the incline is equal to *g* sin*q*. (*Hint:* Draw a free-body diagram!)

2. If the plumb line remains at a fixed angle as the system accelerates, what can be said about:

- the acceleration of the plumb bob as measured in frame *O*¢ *?* Explain.

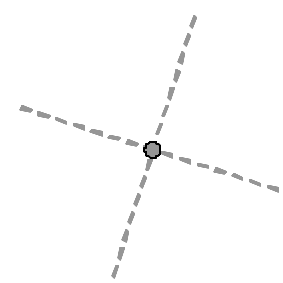
- the sum of all forces on the plumb bob—*physical* *and* *fictitious*—as measured in frame *O*¢ *?* Explain.

3. For the instant described above, carefully draw a free-body diagram for the plumb bob, including all non-zero fictitious "forces." Use the coordinate axes provided as reference.

   (*Big hint:* Draw the tension force by the string **last!**)

   What does your free-body diagram suggest about the orientation of the plumb line relative to the incline? Explain how you can tell.

Free-body diagram for plumb bob *in frame O¢*



   On the basis of your answer, what do you think will be the orientation of the surface of the water inside the container? Explain.

When analyzing a physical situation from a non-inertial reference frame, the actual gravitational force on an object is sometimes added together with the inertial "force." The resultant (vector) sum is often called the *apparent weight* of the object. Dividing an object's apparent weight by its mass yields the *local acceleration due to gravity* ("g_{local}") as measured in the non-inertial frame.

4. For an observer in frame *O¢* (accelerating along with the container-plumb line system):

   - Describe the direction of the apparent weight of the plumb bob relative to the orientation of the incline. (For example, does the apparent weight have a component parallel to the incline?)

   - Is the magnitude of g_{local} *greater than, less than,* or *equal to* the magnitude of *g?* Explain.

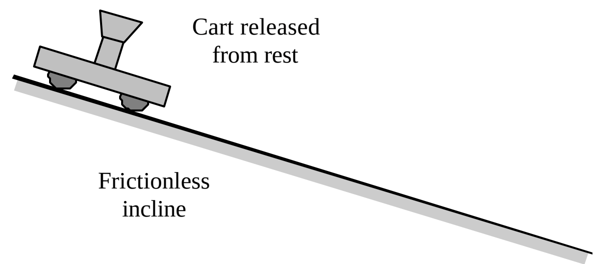
C. On the basis of your results in this section, explain how you can use (i) a plumb line and (ii) a container of (incompressible) fluid to measure the direction of the local acceleration due to gravity. Discuss your reasoning with your partners.

✓ **STOP HERE** and check your results with an instructor before proceeding to the next page.

II. Application: Launch cart on a ramp

A cart is fitted with a mechanism that allows a small ball to be launched from the top of the cart. If the cart remains at rest while the ball is launched, the ball drops back down into the launcher.

Suppose that the cart were released from rest on a long, *frictionless* incline before the ball is launched. (Ignore any effect due to the rotation of the wheels of the cart.)



A. For an instant shortly after the ball is launched, draw free-body diagrams for (i) the cart and (ii) the ball *as measured in the frame of the cart.* Include all appropriate fictitious "forces" on both free-body diagrams.

   (*Hint:* In this case is it appropriate to include an inertial "force" on the free-body diagram for the ball? Why or why not?)

B. *As measured in the frame of the cart*, how does the direction of the local acceleration due to gravity compare to (i) the orientation of the incline? (ii) the direction in which the ball is launched?

   Explain how you can use your free-body diagrams from part A to justify your answer.

C. Would the ball drop *back into* the launcher, *to the left of* it, or *to the right of* it? Explain.

✓ **STOP HERE** and check your results with an instructor.

