

**Pretest: Damped harmonic motion: Energy loss
and the quality factor**

Name _____

A simple harmonic oscillator is set into motion by releasing it from rest at $x = + 1.00$ m.

The oscillator is set into motion once again from the same location, except the oscillator now experiences a retarding force that is linear with respect to velocity. As a result, the oscillator does not return to its original starting position, but instead reaches $x = + 0.80$ m after one period of oscillation.

- A. During the first full oscillation of motion, is it possible to determine what fraction of the oscillator's total energy was dissipated due to the retarding force?

If so: Determine the fraction of the total energy dissipated during the first full oscillation, and explain your reasoning.

If not: State what additional information you would need to answer this question, and explain your reasoning.

- B. When the oscillator finishes a *second* full oscillation, is it possible to predict the maximum displacement of the oscillator?

If so: Determine the maximum displacement after the second full oscillation, and explain your reasoning.

If not: State what additional information you would need to answer this question, and explain your reasoning.

DAMPED HARMONIC MOTION: ENERGY LOSS AND THE QUALITY FACTOR

I. Amplitude of underdamped oscillations

Consider a simple harmonic oscillator (*e.g.*, a mass connected to an ideal spring) that experiences a retarding force that is proportional to the speed of the object. After being released from rest at time $t = 0$, the object is observed to oscillate with period T_d .

The maximum displacement of the oscillator is measured at $t = 0$ and at the end of each of the first three cycles of oscillation (see table at right).

t	$x(t)$
0	20.00 cm
$1T_d$	16.00 cm
$2T_d$	12.80 cm
$3T_d$	10.24 cm

A. As you can see, the maximum displacement does not decrease by the same number of cm with each cycle. However, what *is* true about the manner in which the maximum displacement decreases with each cycle? Discuss your reasoning with your partners.

B. For an underdamped oscillator we can use the function $x(t) = A e^{-g t} \cos(\omega_d t + \phi_0)$ to represent the position of the oscillator as a function of time.

1. Using your results from part A as a guide, determine the value of the quantity $e^{-g T_d}$.

(*Hint:* Use the given functional form of $x(t)$ to write two expressions for $x(t)$, one evaluated at $t = 0$ and the other at $t = T_d = 2\pi/\omega_d$, and compare the expressions to one another.)

2. In your own words, give an *interpretation* for the quantity $e^{-g T_d}$, and explain how your interpretation is consistent with both the tabulated data and the functional form of $x(t)$.

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3. Assuming that the period of the oscillator described above is $T_d = 2.0$ s, determine (a) the value of the damping constant g and (b) the position of the oscillator at $t = 8.0$ s (*i.e.*, $t = 4T_d$). Discuss your reasoning with your partners.

II. Quality factor

An underdamped oscillator loses energy during each oscillation. To describe the rate of energy loss in a damped oscillator, we define a *quality factor* Q that is equal to $2p$ divided by the *fraction of the total energy* that the oscillator loses in a single oscillation.

- A. Consider an underdamped oscillator that is released from rest at $t = 0$. In terms of the damping constant g and period T_d , which are assumed to be known, determine expressions for the following quantities. Discuss your reasoning with your partners.
- the ratio of successive maxima (*i.e.*, the fraction of the amplitude retained by the oscillator after a single cycle)
 - the fraction of total energy retained by the oscillator after a single cycle
(*Hint:* When the oscillator is at a maximum displacement, how does the total energy stored in the oscillator depend on its displacement?)
 - the fraction of total energy lost from the oscillator after a single cycle
 - the quality factor Q of the oscillator

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B. Now consider the case in which the quantity gT_d is much smaller than one ($gT_d \ll 1$).

1. Show that the oscillator described above is not only underdamped but *weakly damped*, i.e., that the damping constant is much smaller than the natural frequency of the oscillator: $g \ll \omega_0$.

2. Extend your results from part A above to show that the quality factor of a weakly damped oscillator can be approximated as $Q \cong \omega_0/2g$ where ω_0 is the angular frequency of the oscillator.

Hint: Use the power series expansion of the exponential function: $e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots$

**Homework: Damped oscillations: Energy loss
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Name _____

1. An ideal (frictionless) simple harmonic oscillator is set into motion by releasing it from rest at $x = + 0.750$ m.

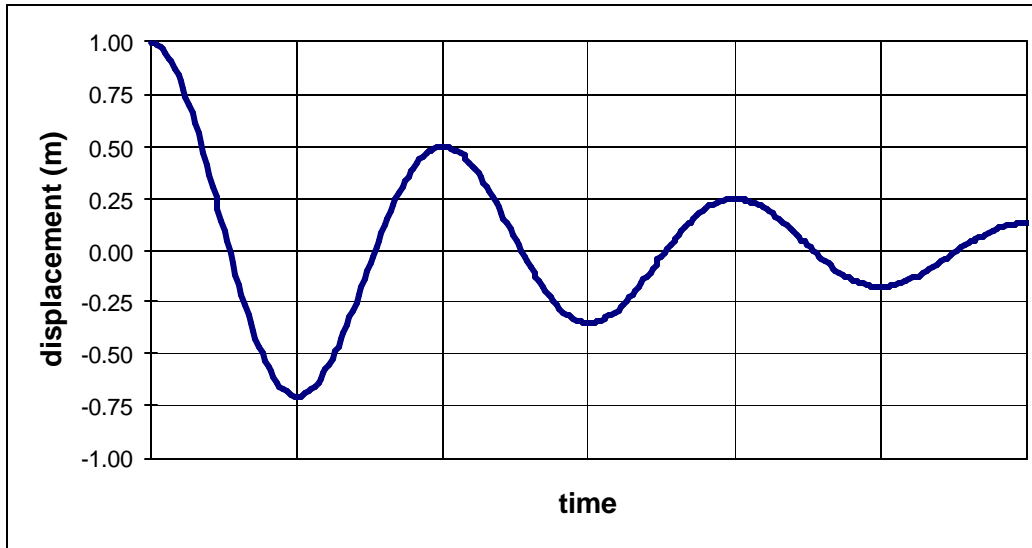
The oscillator is set into motion once again from $x = + 0.750$ m, except the oscillator now experiences a retarding force that is linear with respect to velocity. As a result, the oscillator does not return to its original starting position, but instead reaches $x = + 0.700$ m after one period.

- a. During the first full oscillation of motion, determine the fraction of the oscillator's total energy that was dissipated due to the retarding force. Explain.
 - b. Predict the maximum displacement of the oscillator upon completing its *fourth* full oscillation. Explain your reasoning.
 - c. Determine the quality factor of the oscillator. Show all work.
2. Consider two identical simple harmonic oscillators (1 and 2) to which are applied different damping forces. The two oscillators become underdamped, with the frequency of the first oscillator now greater than that of the second (*i.e.*, $\omega_{d,1} > \omega_{d,2}$).
- a. Which oscillator has the larger damping constant: *oscillator 1, oscillator 2, neither, or is there insufficient information to tell?* Explain your reasoning.
 - b. Which oscillator has the larger quality factor: *oscillator 1, oscillator 2, neither, or is there insufficient information to tell?* Explain your reasoning.
3. Consider a weakly damped oscillator that is released from rest at $t = 0$ and whose amplitude is smaller by a factor of e after undergoing N full oscillations.
- a. Write an expression for the period of oscillation T_d in terms of N and the damping constant g . Explain your reasoning and show all work. (*Hint:* Recall from tutorial how you interpreted the quantity e^{-gT_d} .)
 - b. Do you think that the quality factor Q of this oscillator would be *increases with N , decreases with N , or neither?* Explain *without* performing any calculations.
 - c. Verify your answer in part b above determining an expression for Q in terms of N . Show all work.

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4. The x vs. t graphs in parts a and b below represent the motion of an underdamped oscillator released from rest at $t = 0$.

- a. On the graph below, sketch a qualitatively correct x vs. t graph (drawn to the same scale as the original graph) for another oscillator having the same damping constant g as the original oscillator but a larger quality factor. Explain how you decided to draw the new graph. (*Hint: The frequency of new oscillator may be different from that of the original*)



- b. On the graph below, sketch a qualitatively correct x vs. t graph (drawn to the same scale as the original graph) for an oscillator having the same quality factor as the original oscillator but a smaller damping constant g . Explain how you decided to draw the new graph.

