

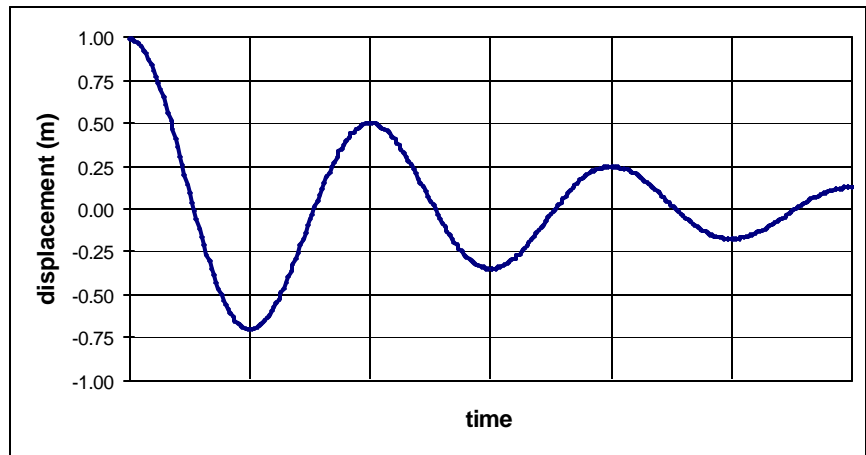
# DAMPED HARMONIC MOTION: MOTION GRAPHS

## I. Displacement versus time

Consider a simple harmonic oscillator (*e.g.*, a mass connected to an ideal spring) that experiences a retarding force that is always proportional to the speed of the oscillator.

At  $t = 0$  the oscillator is displaced 1.0 m away from equilibrium and released from rest. The displacement versus time ( $x$  vs.  $t$ ) graph at right represents the subsequent motion of the oscillator.

(Because the motion still exhibits oscillatory behavior, the oscillator is said to be *underdamped*).



A. According to the graph, how (if at all) does each of the following quantities change as time elapses? Explain how you can tell from the graph.

- the maximum displacement attained with each oscillation
- the period of oscillation

B. Suppose that the retarding force were removed (*e.g.*, the oscillator is now immersed in a vacuum rather than air). Imagine that the oscillator is now released with the *same initial conditions* as before.

How, if at all, would removing the retarding force affect each of the following quantities? Explain your reasoning in each case.

- the average acceleration of the oscillator as it moves from  $x = +1.0$  m to  $x = 0$
- the amount of time required for the oscillator to travel from  $x = +1.0$  m to  $x = 0$

**Damped harmonic motion: Motion graphs**

- C. Your answers in part B above suggest that retarding force affects the period of oscillation. Use your results to predict whether the period of the ideal (frictionless) oscillator is *longer than* or *shorter than* that of the damped oscillator. Explain.

On the graph on the preceding page, illustrate your prediction by sketching a qualitatively correct  $x$  vs.  $t$  graph for the ideal (frictionless) harmonic oscillator. Assume that the initial conditions of the motion are the same as before (*i.e.*, the oscillator is released from rest at  $x = 1.0$  m at  $t = 0$ ).

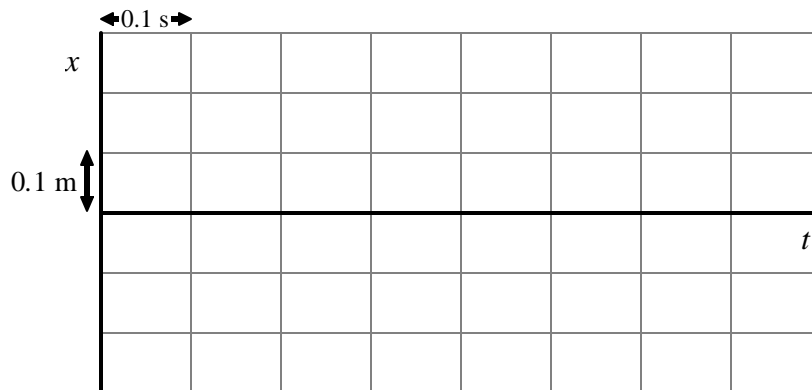
- D. Summarize your results thus far: If an ideal (frictionless) harmonic oscillator is subjected to a retarding force proportional to its speed, how is the frequency of the oscillator affected? Explain.

✓ **STOP HERE** and check your reasoning with an instructor.

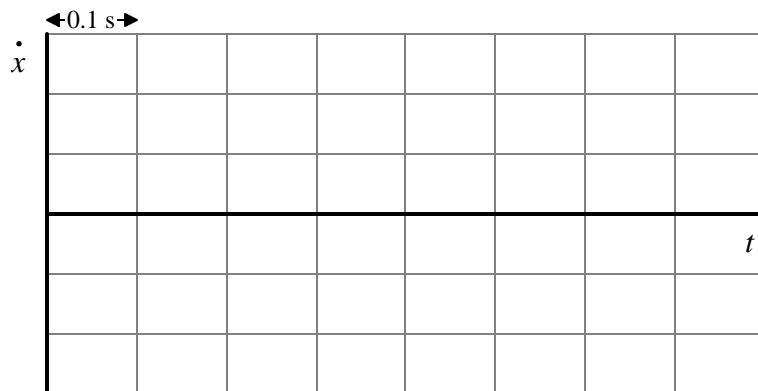
**II. Velocity versus time**

Consider now another underdamped harmonic oscillator with a period of 0.4 s. At  $t = 0$  the oscillator is released from rest at  $x = 0.2$  m.

- A. In the space at right, carefully sketch a position vs. time graph for the oscillator. (Note that each grid corresponds to 0.1 m of distance and 0.1 s of time.)



- B. On the next set of axes, carefully sketch a qualitatively correct graph of velocity vs. time for the oscillator.



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C. Consider the instant (call it  $t_o$ ) at which the oscillator first passes through  $x = 0$ . Identify this instant on both graphs on the preceding page.

1. Is the net force exerted on the oscillator equal to *zero* when it passes through  $x = 0$ ? If not, is the net force in the *same direction* or in the *opposite direction* as the velocity? Explain.

(*Hint:* Draw a free-body diagram for the oscillator for the instant when it passes through  $x = 0$ .)

2. On the basis of your answer in part 1:

- At  $t = t_o$ , is the oscillator moving with *increasing speed*, *decreasing speed*, or *neither*? Explain.

- Does the oscillator first reach a maximum speed *at*  $t = t_o$ , *after*  $t = t_o$ , or *before*  $t = t_o$ ? Explain.

D. Is your velocity vs. time graph from part B (on the preceding page) consistent with your results in part C above? If not, resolve the inconsistencies.

✓ **STOP HERE** and check your results with an instructor.