## INSTRUCTOR NOTES
## *Forced harmonic motion*

### *Emphasis*

Students examine the qualitative behavior of a sinusoidally driven oscillator. They are guided to recognize the significance of steady state motion, including resonance, with regard to the power dissipated by the damping force and the power delivered by the driver. They also apply the concepts of work and power to investigate how the phase difference between the driving force *F(t)* and position *x(t)* affect the amount of power delivered by the driver.

### *Prerequisites*

Students need to have completed instruction of damped oscillations as well as the solution to the equation of motion for sinusoidally driven oscillators. Completion of both tutorials on damped oscillators is recommended but not required for this tutorial.

### TUTORIAL PRETEST

The pretest probes student understanding of two important features of a driven oscillator at resonance: (i) the resonant frequency $w_{\mathrm{r}}$ is never greater than the natural frequency $w_o$ of the oscillator (if both the damping and driving force were removed), and (ii) the sinusoidal driving force *F(t)* leads the position *x(t)* by a phase shift of approximately 90° (or, the driving force is approximately in phase with the velocity of the oscillator).

On the first question, students may recognize that the two oscillators have different frequencies but not realize that the smaller frequency must correspond to the damped, driven oscillator at resonance. Although students may have been shown the equation $w_{\mathrm{r}} = (w_o^2 - 2g^2)^{½}$ for resonant frequency, they do not necessarily recognize that this relationship implies $w_{\mathrm{r}}$ is never greater than $w_o$. On the second question, students may associate the peaks of the driving force with the peak displacements (not the peak velocities) of the oscillator. Such an error may indicate the incorrect belief that the driving force delivers the most power when it is in phase with the position (not velocity) of the oscillator.

### TUTORIAL SESSION

### *Equipment and handouts*

Each group will need a whiteboard and set of markers, or a large sheet of paper. Each student will need a copy of the tutorial handout (no special handouts required).

### *Discussion of tutorial worksheet*

*Section I: Steady-state motion*

This first section of the tutorial gives students the opportunity to interpret in words the meaning of *steady-state* in terms of the total mechanical energy (kinetic plus potential) of the oscillator. It also guides students to think carefully about the motion of the oscillator at (amplitude) resonance. Some students express the intuition that resonance occurs whenever the damping force dissipates the *minimum* (not maximum) amount of power, thinking that resonance implies "minimum resistance" from the damping force. Such a notion, however, together with the fact that the total energy of the oscillator remains constant, would suggest (incorrectly) that at resonance the driving force imparts a *minimum* (not maximum) power to the oscillator. Upon concluding this section students should develop the correct intuition that that resonance requires the maximum power delivered to the oscillator by the driving force. The checkpoint at the end of the section should be used to check student reasoning before continuing with the next section.

*Section II: Phase difference*

In this section of the tutorial students examine in detail how the phase difference $f$ between the driving force $F(t) = F_o \exp(iwt)$ and steady-state position $x(t) = A \exp(iwt - if)$ is related to the power delivered to the oscillator by the driving force. Building upon their conclusions from the preceding section, students investigate under what conditions—specifically, for which value of phase angle *f*—the driving force imparts maximum power to the oscillator.

Students begin the section by interpreting the minus sign in exp*(iwt – if ).* Most readily state that the minus sign means that *x(t)* lags behind *F(t)* by *f*. However, when they are then asked to sketch several *x(t)* graphs for various values of *f* many students struggle. They often explain incorrectly that the minus sign in exp*(iwt – if )* means that they must shift the sinusoidal curve "in the negative *t* direction" (relative to *t* = 0) rather than think of shifting *t* = 0 relative to the sinusoid. (The tutorial *Simple harmonic motion* also attempts to address this type of difficulty.) Watch also for the occasional miscue in relating each given value of *f* to the appropriate fractions of a period (*e.g.*, saying that 45° corresponds to one-quarter of a period rather than one-eighth). The checkpoint in the middle of page 2 is crucial for instructors to make sure that students draw correct *x(t)* graphs before proceeding with the rest of the section.

After correctly drawing the graphs of *x(t)* for the three examples values of *f*, students continue in part C by sketching the corresponding graphs of *v(t).* In part D students should recognize that the power *P* = ***F* • *v*** delivered by the driving force is not the same in all three cases, with *f* = 90° representing the maximum power delivered and *f* = 180° representing the minimum (zero power delivered). Most students recognize as a result that resonance occurs when the driving force is (approximately) in phase with the velocity *v(t)* rather than with the position *x(t)* of the oscillator. Students summarize and apply their findings to answer the questions given in part E.

## TUTORIAL HOMEWORK

The homework gives students the opportunity to apply and extend their results. Most of the extensions guide students to make sense of the mathematics of driven oscillations in light of the qualitative findings obtained in tutorial.

1. Students begin with the expressions for steady-state amplitude *A* = *A(w)* and phase difference *f* = *f (w)*, which are assumed to have been covered previously in class. In part a they are asked to use differentiation in order to find the value of frequency $w = w_r$ that maximizes the amplitude. A hint is provided to help students recognize that, rather than maximize *A(w)* by differentiating the entire expression "brute force" with respect to *w*, it suffices to minimize just the denominator. Students continue by deriving the expression for the phase difference at resonance (in part b) and showing that phase difference at resonance approaches 90°, the result obtained in the tutorial, in the limit of weak damping (in part c).

2. Students revisit the tasks posed on the pretest. Students should complete Problem 1 before attempting this problem; they will need to recognize that the resonant frequency is smaller than the natural (undamped) frequency of the same oscillator. The problem concludes with a quantitative application of the results used previously in the problem. (The calculations also require knowledge of the relationship among the natural frequency, damping constant, and damped frequency of a harmonic oscillator.)

3. In this problem students apply their results from driven mechanical oscillators to driven LRC circuits. In particular, they are guided to recognize the phase relationships between various potential differences in a driven LRC circuit.

4. In the last problem students extend and synthesize results from this tutorial and from their experience with damped oscillators to consider how the transient component of the motion of a driven oscillator combines with the steady-state motion.