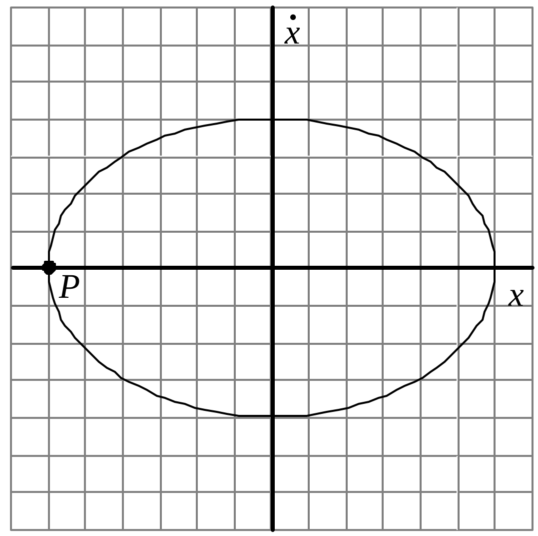
Pretest: Phase space diagrams: Damped harmonic motion Name ___________________________

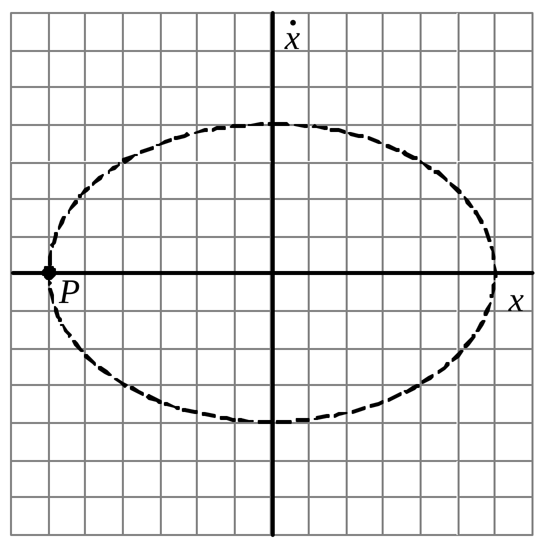
A block, initially at rest at a location along the *negative-x* axis, undergoes simple harmonic motion, as illustrated by the phase space diagram at right. The point labeled *P* denotes the initial conditions for the motion of the block.



Now suppose that the block were released from rest at the <u>same location</u> as before, *except* that a retarding force is now applied to the block. Assume that the retarding force is proportional to the velocity of the block.

a. Consider the case in which the retarding force causes the block to undergo *underdamped* motion.

   i. In the space at right, sketch how the phase space trajectory of the block would differ from the original trajectory (shown as a dashed curve). Explain your reasoning.

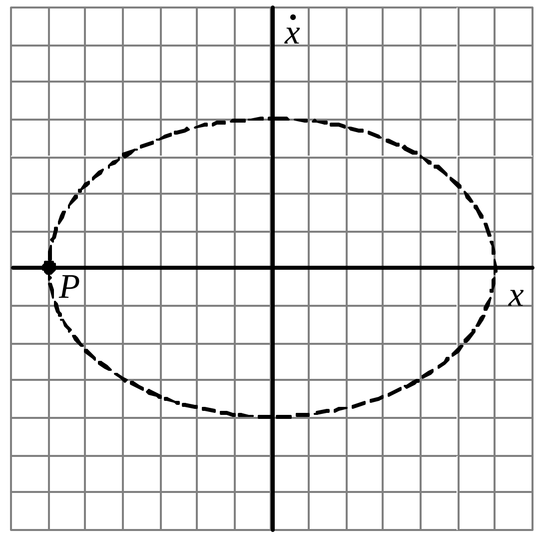


   ii. Consider the first moment after release when the block would attain a maximum speed. At that moment would the block be located *to the left of* $x = 0$, *to the right of* $x = 0$, or *exactly at* $x = 0$? Explain.

(continued on other side)

b. Consider instead the case in which the retarding force is great enough to cause the block to undergo *critically damped* motion.

In the space at right, sketch how the phase space trajectory of the block would differ from the original trajectory (shown as a dashed curve). Explain your reasoning.



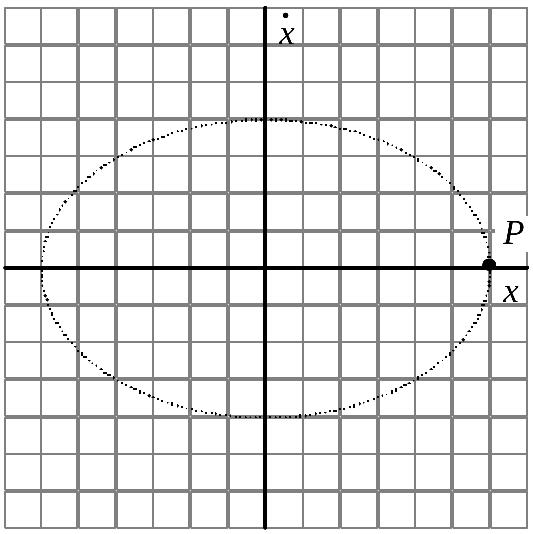
PHASE SPACE DIAGRAMS: DAMPED HARMONIC MOTION

I. Underdamped oscillator

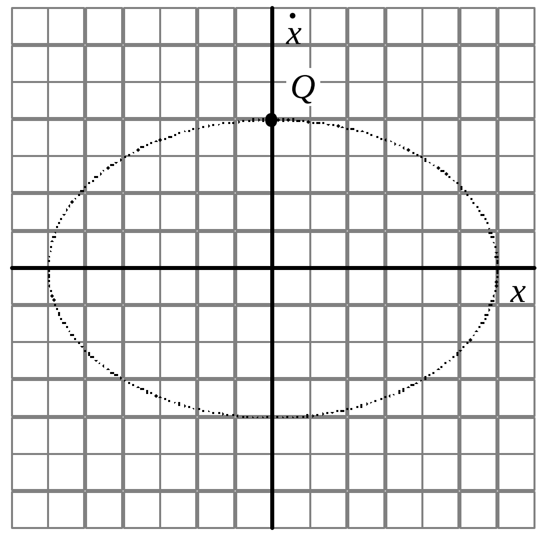
Suppose that a simple harmonic oscillator were subject to a retarding force that is proportional to the velocity of the oscillator.

Each phase space plot shown below corresponds to a motion of the oscillator in the undamped case.

A. On each diagram, sketch the (approximate) phase space trajectory for the situation described under each plot. Discuss your reasoning with your partners.



Case 1: Starts at point *P;* amplitude decreases by a factor of 2 with each oscillation



Case 2: Starts at point *Q;* amplitude decreases by a factor of 4 with each oscillation

B. For a damped oscillator, is it correct for the phase space trajectory to cross the vertical $(\dot{x})$ axis at right angles? Explain why or why not. (*Hint:* In this case, is the net force exerted on the oscillator equal to zero when it passes through $x = 0$?)

For a damped oscillator, is it correct for the phase space trajectory to cross the horizontal *(x)* axis at right angles? Explain why or why not.

C. Are the phase space trajectories that you sketched in part A consistent with your answers in part B? If not, resolve the inconsistencies.

✓ **STOP HERE** and check your results with an instructor before proceeding to the next section.

II. Critically damped oscillator

Now suppose that the oscillator were critically damped, *i.e.*, suppose that the damping factor (g) for the retarding force were now equal to the angular frequency of the undamped oscillator $(g = w_o)$. In this case, the position $x(t)$ of the critically damped oscillator is given by:

$$x(t) = (At + B)\, e^{-gt}$$

where A and B are arbitrary constants.

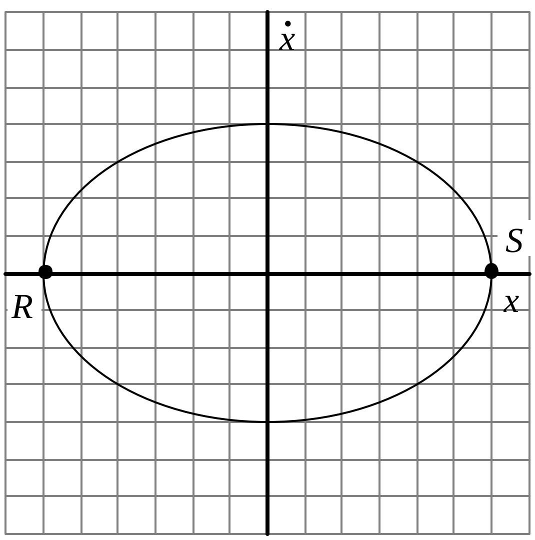
A. Differentiate the above expression for $x(t)$ and show that the parametrized equation $\dot{x}(x, t)$ can be written:

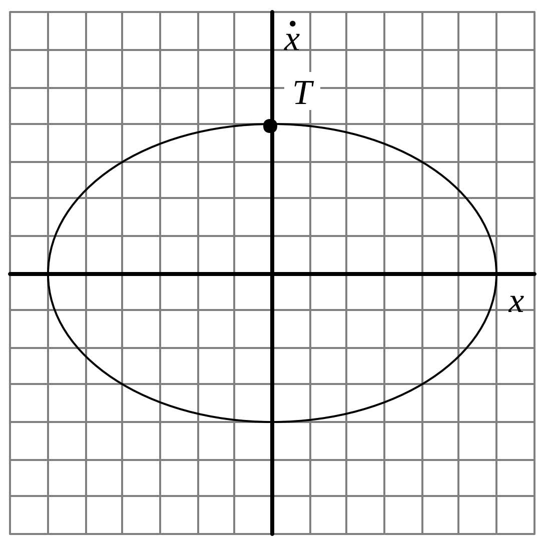
$$\dot{x} = -\,gx + A\, e^{-gt}$$

Your result above suggests that the asymptotic behavior (as $t \to \infty$) of the critically damped oscillator can be represented by a *straight line* on a phase space diagram. What is the equation for this line?

B. Each phase space plot shown below corresponds to a motion of the oscillator in the undamped case.

1. On each diagram, *accurately* sketch the line that would describe the asymptotic behavior of the oscillator in the critically damped case. Explain your reasoning.





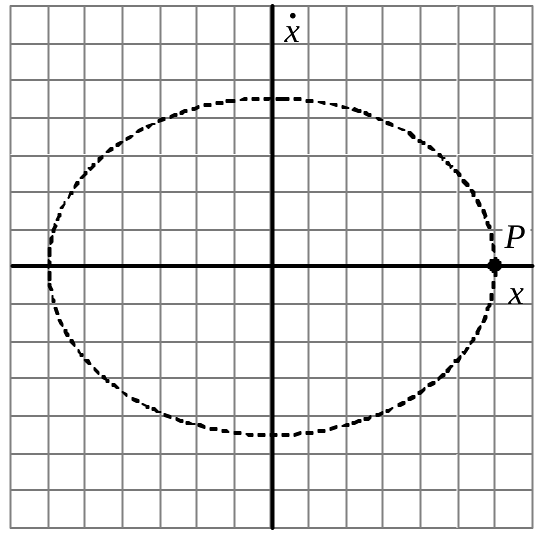
2. For each of the starting points *(R, S,* and *T)* shown in the diagrams above, draw a qualitatively correct phase space trajectory for the subsequent motion of the critically damped oscillator. Discuss your results with your partners.

✓ **STOP HERE** and check your results with an instructor.

Homework: Phase space diagrams: Damped harmonic motion Name ______________________

1. The phase space trajectory of an undamped oscillator is shown below right. In the diagram, each division along the position axis corresponds to 0.1 m; along the velocity axis, 0.10 m/s.

   a. What is the angular frequency w_o of the undamped oscillator? Explain how you can tell.



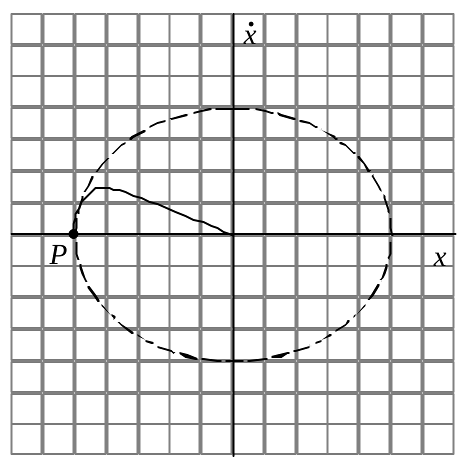
   A retarding force is now applied to the oscillator for which the damping constant is equal to $g = 0.069w_o$.

   b. By what factor does the amplitude change after a single oscillation? Show all work.

   c. On the basis of your results above, carefully sketch the phase space plot for the <u>first cycle</u> of the motion of the damped oscillator, starting at point *P*.

2. Shown below right are the phase space plots for (i) a simple harmonic oscillator (dashed) and (ii) the same oscillator with a retarding force applied (solid). Point *P* represents the initial conditions of the oscillator in both instances.

   a. Explain how you can tell that the damped oscillator is *not* underdamped.



   b. Is the damped oscillator *critically damped* or *overdamped?* Explain how you can tell.

   c. If you said in part b that the oscillator is {critically damped, overdamped}, then draw how the phase space plot would be different if the oscillator (starting at point *P*) were instead {overdamped, critically damped}. Explain your reasoning.