

1. The potential energy function for a 2-D oscillator may be written: $U(x, y) = \frac{1}{2} k_1 x^2 + \frac{1}{2} k_2 y^2$.

Show that component differential equations of motion are *separable*, and explain why the position of the oscillator can be written as:

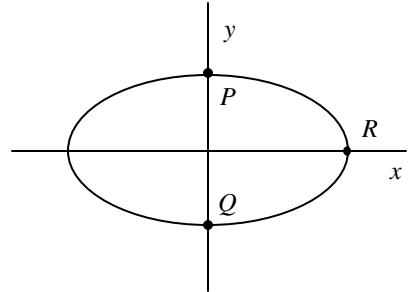
$$x(t) = A_1 \cos(\omega_0 t + \mathbf{j}); \quad y(t) = A_2 \cos(\omega_0 t + \mathbf{j} + \mathbf{d})$$

2. Shown at right is the x - y trajectory for a 2-D oscillator.

- a. Consider the following *incorrect* statement:

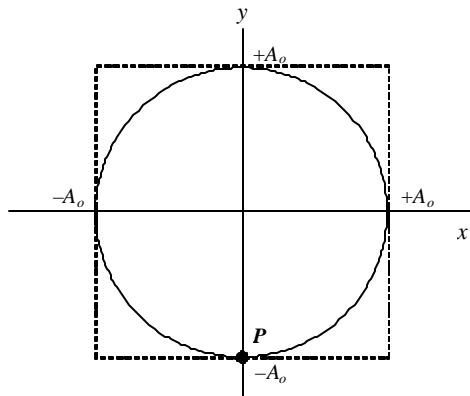
“The oscillator goes farther in the x -direction than in the y -direction. That means the spring in the y -direction must be stiffer than the spring in the x -direction.”

Identify the error in the above statement, and state how you would modify it to make it correct.

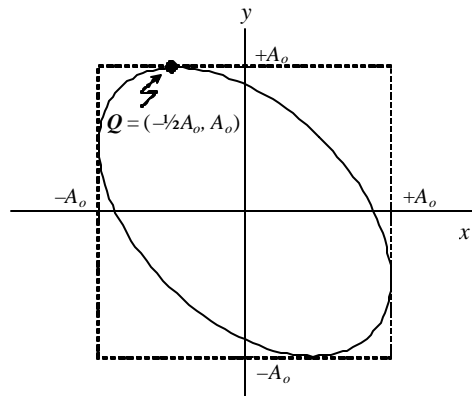


- b. Rank the labeled points (P , Q , and R) according to: (i) total energy, (ii) potential energy, (iii) kinetic energy. Explain how the difference in amplitudes in the x - and y -directions, used *incorrectly* in the statement in part a, can be used to justify a *correct* conclusion in this part of the problem.

3. Each diagram below describes the motion of a 2-D harmonic oscillator.



Case #1: Oscillator starts at P with initial speed $5A_0$ m/s to the right



Case #2: Oscillator starts at Q , goes clockwise, and returns to Q 4π sec later

For each case, determine the exact expressions $x(t)$ and $y(t)$ that describe the position of each oscillator as functions of time. (Use the functional forms for $x(t)$ and $y(t)$ shown in Problem 1 of this homework.) Explain your reasoning and show all work.

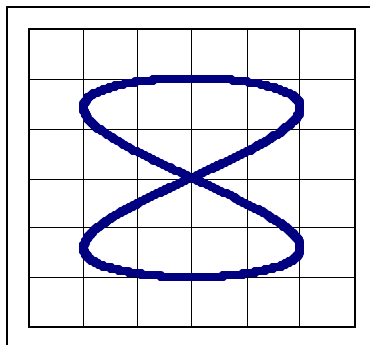
Homework: Harmonic motion in two dimensions

4. Consider the motion of a two-dimensional oscillator, $U(x, y) = \frac{1}{2} k_1 x^2 + \frac{1}{2} k_2 y^2$, in which the spring constants are *unequal*: $k_1 \neq k_2$.
- a. Justify the term *non-isotropic* as it applies to such an oscillator.

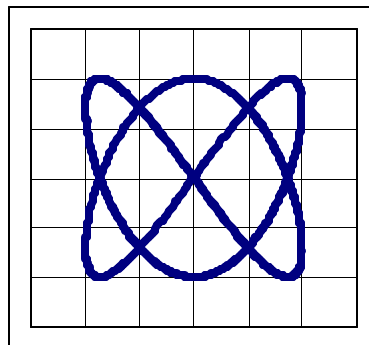
Each trajectory below depicts the possible motion of a unique oscillator. All three oscillators, however, share the property that the angular frequencies ω_1 and ω_2 for the motions along the x - and y -axes are *commensurate*, i.e., that the angular frequencies satisfy the following relationship:

$$\frac{\omega_1}{n_1} = \frac{\omega_2}{n_2}, \text{ where } n_1 \text{ and } n_2 \text{ are integers.}$$

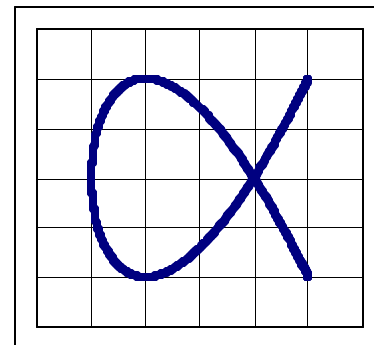
- b. For each case below, (i) determine whether ω_1 is *greater than*, *less than*, or *equal to* ω_2 , and (ii) determine the values n_1 and n_2 that satisfy the condition shown above. Explain.



Trajectory #1



Trajectory #2



Trajectory #3

- c. In **Trajectory #1** above, suppose that the oscillator has a mass of 0.20 kg and retraces its path every 4π seconds. Determine the numerical values of ω_1 , ω_2 , k_1 and k_2 for that case. Show all work.