HARMONIC MOTION IN TWO DIMENSIONS

I. Frequencies of motion

A small block of mass *m* is attached to a massless spring (spring #1) with force constant k_1 and placed upon a frictionless horizontal surface. The block undergoes simple harmonic motion in a straight line.

A. If you wished to double the frequency of oscillation (using the same block), (i) would you *increase* or *decrease* the force constant, and (ii) by *what factor* would you change the force constant? Discuss your reasoning with your partners.

Now imagine that another spring (spring #2) with force constant k_2 is attached to the block so that the block can undergo oscillatory motion simultaneously in orthogonal directions.

Note: We will assume that spring #1 is always parallel (or essentially parallel) to the *x*-axis, and that spring #2 is always parallel (or essentially parallel) to the *y*-axis.

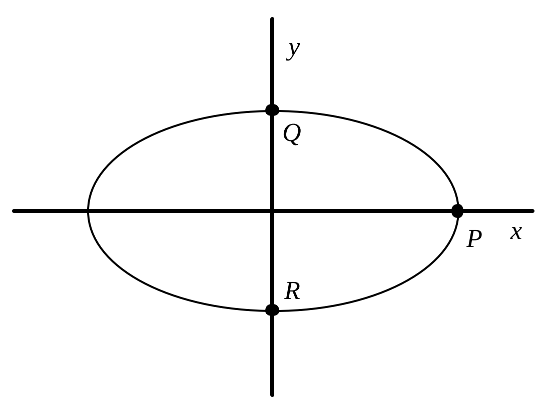
B. If the force constants k_1 and k_2 were equal to each other, how would the frequencies of motion along the *x*- and *y*-axes compare to one another? Explain your reasoning.

   How would the frequency of oscillations along the *x*-axis compare to that along the *y*-axis if instead the force constants were *unequal, e.g.,* if $k_1 > k_2$? Explain.

C. The diagram below right shows the *x*-*y* trajectory of an example 2-D oscillator. The amplitude of motion along the *x*-axis is larger than that along the *y*-axis.

   1. For each period of oscillation along the *x*-axis, how many periods of oscillation occur along the *y*-axis? Explain how you can tell.

   2. For the 2-D oscillator described here in part C, is k_1 *greater than, less than,* or *equal to* k_2? Explain how you can use your results from parts A and B above to support your answer.



x-*y* trajectory of a 2-D oscillator

II. Trajectories of 2-D isotropic oscillators

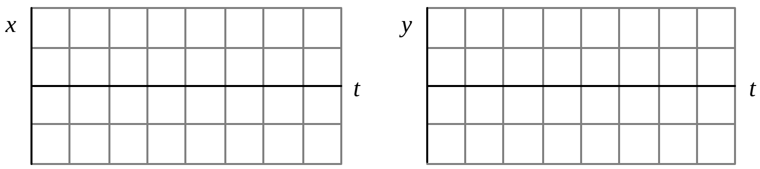
For the remainder of this tutorial, consider an *isotropic* oscillator, *i.e.*, consider a 2-D oscillator for which the force constants are equal: $k_x = k_y = k$. The positions *x(t)* and *y(t)* of the oscillator can be written:

$$x(t) = A_1 \cos(w_o t + j); \quad y(t) = A_2 \cos(w_o t + j + d)$$

A. In the expressions for *x(t)* and *y(t)* presented above, explain why *two* phase angles (*j* and *d*) are needed in order to make those expressions as general as possible.

B. The *x-y* trajectory shown in section I (on the preceding page) can result from many possible initial conditions of motion. For each set of initial conditions listed below, (i) sketch qualitatively correct graphs of *x(t)* and *y(t)*, and (ii) state the appropriate values of the phase angles *j* and *d*.

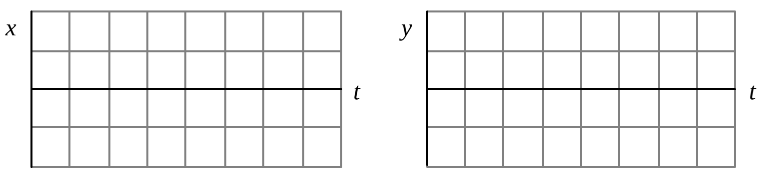
- Initial position at point *P*, initial velocity in +*y* direction



j =

d =

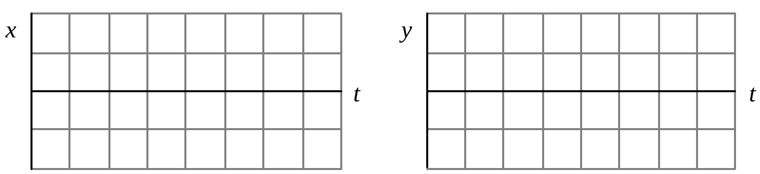
- Initial position at point *P*, initial velocity in –*y* direction



j =

d =

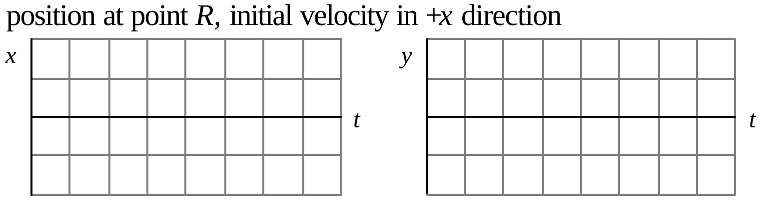
- Initial position at point *Q*, initial velocity in +*x* direction



j =

d =

- Initial position at point *R*, initial velocity in +*x* direction



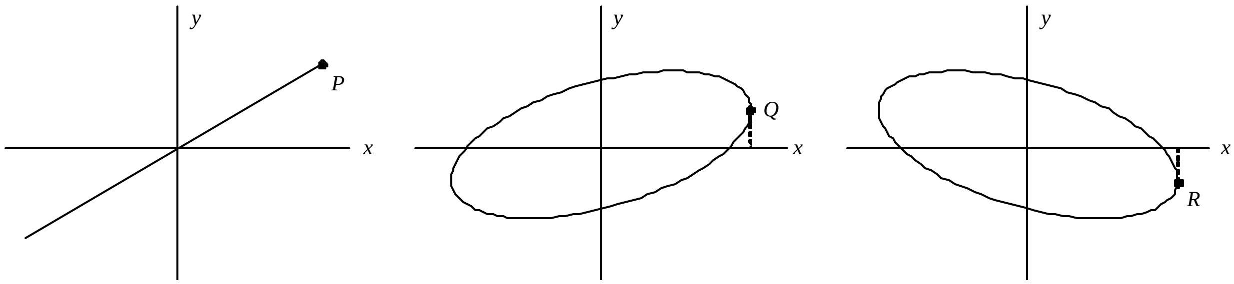
j =

d =

C. Generalize your results thus far: When an isotropic oscillator in two dimensions follows an elliptical trajectory whose axes coincide with the *x-y* coordinate axes:

- What is the value of |*d* |?
- What is the sign of *d* if the oscillator follows the trajectory clockwise? Counter-clockwise?

D. Shown below are several possible trajectories for a 2-D isotropic oscillator whose axes do *not* coincide with the *x*-*y* coordinate axes.



For each of the four cases described below:

- Deduce whether the phase angle ***d*** is *positive, negative,* or *zero* for that case.
- If ***d*** is non-zero, determine whether the absolute value of the phase angle |***d***| is *between zero and 90°*, *equal to 90°*, *between 90° and 180°*, or *equal to 180°* for that case.

1. The oscillator follows trajectory #1 by starting from rest at point *P*.

2. The oscillator follows trajectory #2 by starting from point *Q* and proceeding clockwise.

3. The oscillator follows trajectory #2 by starting from point *Q* and proceeding counter-clockwise.

4. The oscillator follows trajectory #3 by starting from point *R* and proceeding clockwise.

E. Generalize your results: Describe how varying the phase angle ***d*** from –180° to +180° affects the motion of a 2-D isotropic oscillator. In particular, how would you characterize the <u>shape of the trajectory</u> or the <u>direction of motion</u> when the phase angle ***d***:

- is *positive* in value? is *negative* in value?
- is equal to *0°?* is equal to *180°?*
- has absolute value *between 0° and 90°?* has absolute value *between 90° and 180°?*

✓ **STOP HERE** and check your results with an instructor.

