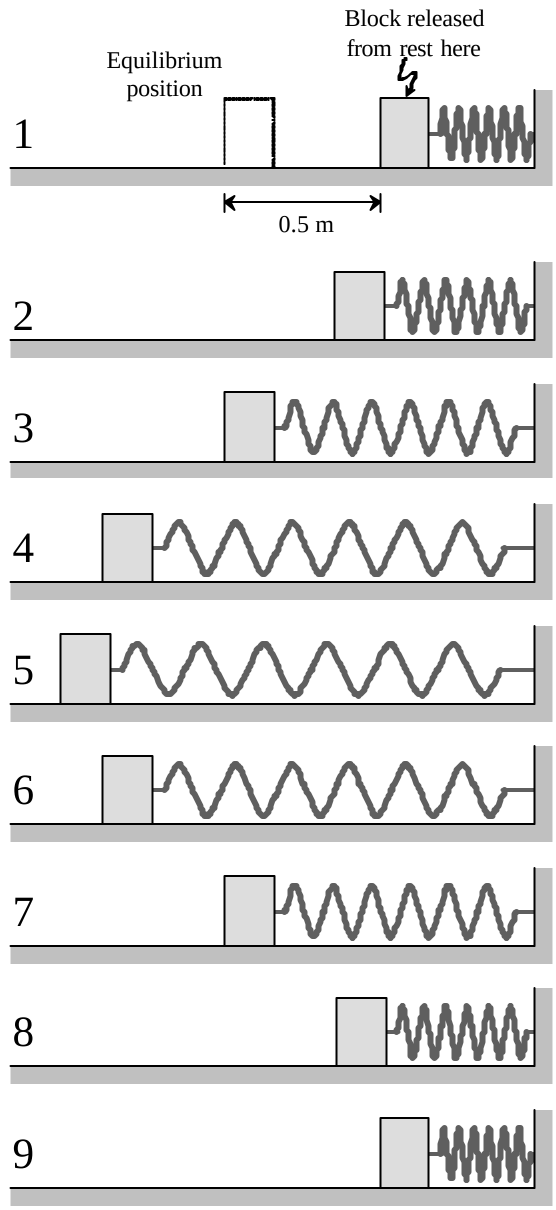
SIMPLE HARMONIC MOTION

I. Qualitative analysis of motion

A block is connected to a spring, one end of which is attached to a wall. (Neglect the mass of the spring, and assume the surface is frictionless.)

The block is moved 0.5 m to the right of equilibrium and released from rest at instant 1. The strobe diagram at right shows the subsequent motion of the block (*i.e.*, the block is shown at equal time intervals).



A. For each instant, draw a vector that represents the *instantaneous velocity* of the block at that instant. Explain how you decided to draw your vectors.

B. Use your velocity vectors from part A to determine graphically the direction of the *average acceleration* ($\vec{a} \equiv \Delta\vec{v}/\Delta t$) from instant 2 to instant 3. Discuss your reasoning with your partners.

Modify your approach as necessary to determine:

- the direction of the *average acceleration* from instant 4 to instant 5, and from instant 5 to instant 6

- the direction of the *instantaneous acceleration* at instants 2, 4, and 5

Are your results above consistent with your knowledge of forces and Newton's laws? If so, explain why. If not, resolve the inconsistencies.

✓ **STOP HERE** and check your results with an instructor.

II. Differential equation of motion

Consider again the situation depicted in section I, in which a block of mass *m* attached to an (ideal) spring of force constant *k* undergoes simple harmonic motion on a level, frictionless surface.

A. Using Newton's second law in one dimension, $F_{net} = m\ddot{x}$, write down the differential equation that governs the motion of the block.

The net force exerted on the block may be called a *restoring force*. Justify this term on the basis of your differential equation above.

B. Show by direct substitution that the following functions are solutions to the differential equation you wrote down in part A. As part of your answer, specify the conditions (if any) that must be met by the parameters A, w, and f_o in order for each function to be a valid solution.

- $x(t) = A\cos(wt + f_o)$
- $x(t) = A\sin(wt + f_o)$

C. Suppose that the experiment described in section I were repeated exactly as before, except with one change to the setup. For each change described below, how (if at all) would that change affect the period of motion? Be as specific as possible, and use your results from part B to justify your answers.

1. The spring is replaced with a stiffer spring.

2. The block is replaced with another block with four times the mass as the original one.

3. The block is released 0.3 m (instead of 0.5 m) to the right of equilibrium.

✓ **STOP HERE** and check your results with an instructor.

III. Expressing position as a function of time

A. Consider again the motion of the block in section I, including the initial conditions of the motion. For each of the functions you examined in part B (see below), evaluate all constant parameters $(A,\ w,$ and $f_o)$ so as to completely describe the position of the block as a function of time.

- $x(t) = A\cos(wt + f_o)$

- $x(t) = A\sin(wt + f_o)$

B. Check your answers in part A above by examining the dialogue below.

Chris: "The cosine function is the same as a sine curve that has been shifted along the time axis to the left by $p/2$ radians."

Pat: "That's right. That means that the function cos wt is identical to $\sin(wt - p/2)$, because the phase shift of $-p/2$ shifts the sine curve to the left by $p/2$."

Chris' statement is *correct*, however Pat's response is *incorrect*. Identify the error in Pat's reasoning and describe how you would modify Pat's statement so that it would be correct.

C. Are your answers in parts A and B consistent with each other? If not, resolve the inconsistencies.