

IMPORTANT: In answering the following questions, do **not** neglect air resistance!

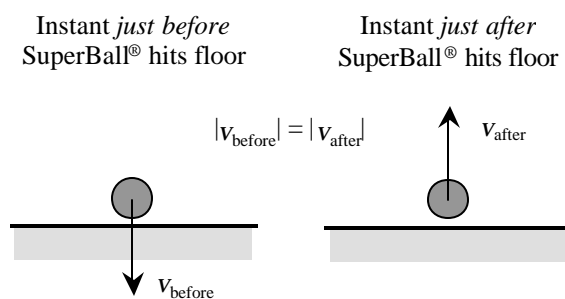
1. You are attending the Muskegon Air Show with your best friend when you watch a skydiver jump from a plane. The skydiver falls freely for a short while before opening her parachute. Soon after the chute has opened, the skydiver descends with constant speed.
 - a. In the space provided below, draw separate free-body diagrams for the system consisting of the skydiver and parachute:
 - i. immediately after the diver has jumped out of the plane, and
 - ii. shortly after the skydiver (with parachute open) begins to move with constant speed.

Clearly label all forces on both free-body diagrams.

- b. Is the net force on the skydiver in case (i) *greater than*, *less than*, or *equal to* the net force on the skydiver in case (ii)? Explain.

2. A SuperBall[®] is dropped to the floor and bounces back straight up. Suppose that the upward speed of the SuperBall immediately after leaving the floor were exactly equal to its downward speed immediately before it reaches the floor.

(Do *not* neglect air resistance in this situation. However, assume that the SuperBall never reaches terminal speed.)



At which instant is the acceleration of the SuperBall *larger* in magnitude: (i) just before it reaches the floor, (ii) just after it leaves the floor, or (iii) is it the same at both instants? Explain.

NEWTON'S LAWS AND VELOCITY-DEPENDENT FORCES

I. Air resistance and Newton's second law

Suppose that you took a small rubber ball to the top of a very tall building and dropped it from rest at $t = 0$. At a later time $t = t_o$, the ball moves with *constant speed*. (The constant speed eventually reached by the falling ball is called the *terminal speed*.)

- A. In the space provided below, draw separate free-body diagrams for the ball (i) shortly after release, and (ii) shortly after it has reached terminal speed. Clearly label all forces.

What can be said about the acceleration of the ball (i) at $t = 0$? (ii) at $t = t_o$? Discuss both magnitude and direction. Explain how you can use your free-body diagrams above to support your answers.

- B. In the space at right, sketch a qualitatively correct graph of velocity vs. time (v vs. t) for the ball. On your graph, clearly label the instant $t = t_o$ on the horizontal axis.



On the same set of axes above, show the v vs. t graph that *would have been* correct if there were *no* air resistance. Make sure your graph is consistent with your first v vs. t graph.

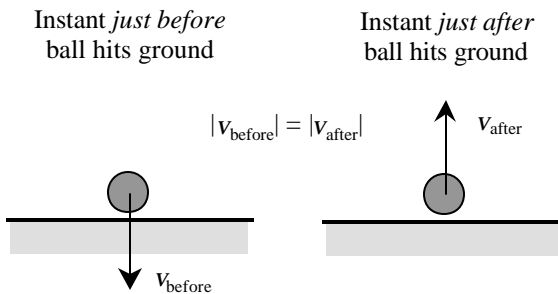
Describe in words how you made sure that both v vs. t graphs were consistent with each other.

- ✓ **STOP HERE** and check your results with an instructor before proceeding to the next section.

Newton's laws and velocity-dependent forces

- C. Now imagine releasing the rubber ball at shoulder level. Suppose that the upward speed of the ball immediately after it leaves the ground were exactly equal to its downward speed immediately before it reaches the ground.

(Assume the ball never reaches terminal speed, but continue to take air resistance into account.)



1. Consider the following conversation between two students:

Student 1: "Acceleration is derived from velocity, which is equal in magnitude at both instants. That means that the ball has the same acceleration at both times."

Student 2: "That's right. In fact, if the ball has the same speed at both instants, then the force of air resistance is the same as well, so the net force is the same at both instants."

In the space below, write down whether you *agree* or *disagree* with each student.

2. Draw separate free-body diagrams for the ball (i) immediately before it reaches the ground, and (ii) immediately after it leaves the ground. Clearly label each force.

On the basis of your results, at which instant is the acceleration of the ball *larger* in magnitude: (i) just before it reaches the ground, (ii) just after it leaves the ground, or (iii) is it the same at both instants? Explain your reasoning.

3. Refer again to the two statements from part 1. Do you agree or disagree with each statement?

If you disagree with any of the student statements, identify the specific error in reasoning used by that student. Discuss your reasoning with your partners.

✓ **STOP HERE** and check your results with an instructor before proceeding to the next section.

Newton's laws and velocity-dependent forces

For the remainder of these exercises we will consider the motion of spherical objects that fall vertically in air (or some other viscous fluid). In such a case, the force of air resistance is treated as being

(i) proportional to the speed of the object (with magnitude c_1v), (ii) proportional to the square of the speed (with magnitude c_2v^2), or (iii) a combination of the above two terms (linear and quadratic).

II. Calculating terminal speed

In answering the following questions, let vertically downward be defined as the positive direction.

A. Suppose that a small ball were moving with downward velocity v .

Starting with Newton's 2nd law, write an equation that includes the acceleration dv/dt of the ball and all relevant force terms (mg , c_1v , and c_2v^2). In particular, carefully decide the sign (whether “+” or “-”) that belongs in front of each individual term. Discuss your reasoning with your partners.

B. How would your equation in part A be different if the ball were instead moving upward? Explain. (*Hint:* What is the sign of v in this case, and how must your equation change as a result?)

✓ **STOP HERE** and check your results with an instructor before continuing.

C. If the force of air resistance exerted were purely *linear* with respect to velocity ($c_1 \neq 0$, $c_2 = 0$), use the appropriate equation to express the terminal velocity v_t of the object in terms of c_1 , m , and g .

Check that your expression for v_t has the correct units. That is, determine the appropriate units for c_1 and confirm that your expression for v_t indeed has the appropriate units for a speed.

1. Consider a ball that moves vertically under the influences of both gravity and air resistance. For the purposes of this problem, take vertically *upward* as the *positive* direction. (For instance, the gravitational force on the ball would be expressed as $-mg$ in this case.)

For each equation of motion below, determine whether that equation applies to (a) a situation in which the ball moves *upward*, (b) a situation in which the ball moves *downward*, (c) *either* of these, or (d) *neither* of these. Explain your reasoning for each case.

i. $m (dv/dt) = -mg + c_1v$

iii. $m (dv/dt) = -mg + c_2v^2$

ii. $m (dv/dt) = -mg - c_1v$

iv. $m (dv/dt) = -mg - c_2v^2$

2. Follow the same reasoning that you used in section II of the tutorial by expressing the terminal velocity of an object of mass m for the case in which the force of air resistance is:

- *quadratic* with respect to speed ($c_1 = 0, c_2 \neq 0$)
- is expressed as a combination of *both* linear *and* quadratic terms ($c_1 \neq 0, c_2 \neq 0$)

NOTE: For problems 3 and 4, use the fact that the force of air resistance on a spherical object of diameter D can be approximated using coefficients $c_1 = (1.55 \times 10^{-4})D$ and $c_2 = 0.22D^2$ (all numerical values are in SI units).¹ The ratio of the quadratic and the linear terms of the force of air resistance can therefore be expressed as:

$$\left| \frac{c_2 v^2}{c_1 v} \right| = \frac{0.22 v |v| D^2}{(1.55 \times 10^{-4}) v D} = (1.4 \times 10^3) |v| D$$

3. Consider a softball with diameter 10 cm and mass 200 g.
- Using the above ratio, for what range of speeds will (i) the linear term of air resistance dominate over the quadratic term? (ii) the quadratic term dominate over the linear term?
 - Calculate the terminal speed of the softball taking into account *both* the linear *and* quadratic terms. Show all work.
 - Reflect on your results in parts a and b above. If it were desired to approximate the effect of air resistance on a falling softball with *either* the linear term *or* the quadratic term (not both), which term would you keep? Explain your reasoning.
4. Repeat problem 3, except now consider an oil droplet from Millikan's oil-drop experiment (use $D = 10^{-4}$ cm, mass 10^{-12} g). Explain your reasoning and show all work.

¹ From *Analytical Mechanics*, 7th ed., Fowles & Cassiday (Thomson-Brooks/Cole), p. 69.