PLASTICITY:
RESOURCE JUSTIFICATION
AND DEVELOPMENT

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An Abstract of the Thesis Presented
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Physics education research is fundamentally concerned with understanding the processes of student learning and facilitating the development of student understanding. A better understanding of learning processes and outcomes is integral to improving said learning. In this thesis, I detail and expand upon Resource Theory, allowing it to account for the development of resources and connecting the activation and use of resources to experimental data.

Resource Theory is a general knowledge-in-pieces schema theory. It bridges cognitive science and education research to describe the phenomenology of problem solving. Resources are small, reusable pieces of thought that make up concepts and arguments. The physical context and cognitive state of the user determine which resources are available to be activated; different people have different resources about different things. Over time, resources may develop, acquiring new meanings as they activate in different situations. In this thesis, I introduce “plasticity,” a continuum for describing the development of resources.
The plasticity continuum blends elements of Process/Object and Cognitive Science with Resource Theory. The name evokes brain plasticity and myelination (markers of learning power and reasoning speed, respectively) and materials plasticity and solidity (with their attendant properties, deformability and stability). In the plasticity continuum, the two directions are *more plastic* and *more solid*. More solid resources are more durable and more connected to other resources. Users tend to be more committed to them because reasoning with them has been fruitful in the past. Similarly, users tend not to perform consistency checks on them any more. In contrast, more plastic resources need to be tested against the existing network more often, as users forge links between them and other resources.

To explore these expansions and their application, I present several extended examples drawn from an Intermediate Mechanics class. The first extended example comes from damped harmonic motion; the others discuss coordinate system choice for simple pendula. In every case, the richness of student reasoning indicates that a wealth of resources of varying plasticity are in play. To analyze the encounters, a careful and fine-grained theoretical approach is required.
ACKNOWLEDGEMENTS

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My husband Matt supports me in all things, and I would not have written this long and occasionally tedious document had he not pushed me out of the house to write. I was grumbly then, but I’m grateful now.
# TABLE OF CONTENTS

ACKNOWLEDGEMENTS ........................................ ii

LIST OF TABLES ............................................. vi

LIST OF FIGURES ........................................... vii

Chapter

1 INTRODUCTION ........................................... 1

2 THEORETICAL FRAMEWORKS ................................. 4

2.1 Metaphors we’ve learned by ............................ 4

2.2 Five traditions: an overview .......................... 8

2.2.1 Cognitive science ................................... 9

2.2.2 Ecological Approach ............................... 13

2.2.3 Conceptions .......................................... 15

2.2.4 Pieces .............................................. 18

2.2.5 Process/Object ...................................... 19

2.3 Resource Theory ......................................... 22

2.3.1 Resources, the units of Resource Theory ......... 23

2.3.1.1 Kinds of resources .......................... 23

2.3.1.2 Two states ...................................... 24

2.3.1.3 Connection schemes .......................... 25

2.3.1.4 Internal structure ............................. 26

2.3.1.5 Notation conventions .......................... 27

2.3.2 Resource Heuristics ................................. 27

2.3.3 Resource Theory strengths ........................ 29

2.3.4 Open questions in Resource Theory .............. 29

3 PLASTICITY .................................................. 31

3.1 Introducing Plasticity .................................. 32

3.1.1 Comparing plastic and solid ..................... 33

3.1.2 The RBC model for abstraction .................. 35
3.2 Plasticity heuristics ............................................. 36
  3.2.1 Frames and Framing ........................................ 38
    3.2.1.1 A note on terminology ................................ 38
    3.2.1.2 Knowledge schema and framing ...................... 39
  3.2.2 Framing and Plasticity ...................................... 40
3.3 Heuristics in action: an example ............................... 41
  3.3.1 Forcesign .................................................... 42
  3.3.2 Plasticity Analysis ......................................... 43
    3.3.2.1 Request for reasoning ................................. 44
    3.3.2.2 Sense-making: elaboration ......................... 44
    3.3.2.3 Sense-making: consistency check ................... 45
    3.3.2.4 Justification through activity .................... 45
    3.3.2.5 Social norm: agreement ............................. 45
  3.3.3 Extended use .............................................. 46
  3.3.4 Discussion .................................................. 48
3.4 Toulmin’s argumentation structure ................................ 48
3.5 Some limitations of plasticity ................................ 52
3.6 Summary ....................................................... 54

4 COORDINATE SYSTEMS ............................................. 55
  4.1 Unpacking coordinate systems ................................. 56
  4.2 Coordinate Systems in Intermediate Mechanics ............... 59
  4.3 Simple Pendulum ............................................... 60
    4.3.1 A physicist’s solution .................................. 62
    4.3.2 The students’ problems ................................. 63
    4.3.3 Summary of common resource use ...................... 64
    4.3.4 A physicist’s graph? .................................... 65
  4.4 Miniviews ...................................................... 67
    4.4.1 Defining a coordinate system, Week 4 ................ 68
    4.4.2 Position, time, and span, Week 4 .................... 72
    4.4.3 Origin .................................................... 78
    4.4.4 Revisiting polar coordinates, Week 10 ............... 80
    4.4.5 Discussion ................................................ 83
  4.5 Dialing down the scale: HHS .................................. 85
4.5.1 Participants ......................................................... 86
4.5.2 Coordinates-1 .................................................... 87
  4.5.2.1 Choosing a system ........................................... 87
  4.5.2.2 Choosing \( \hat{\theta} \) the first time ..................... 89
  4.5.2.3 Choosing \( \hat{r} \) ............................................. 91
  4.5.2.4 Finding zero .................................................. 91
  4.5.2.5 Testing Coordinates ....................................... 93
  4.5.2.6 An analogy to Cartesian .................................. 95
  4.5.2.7 Summary ..................................................... 98
4.5.3 R-forces .......................................................... 99
  4.5.3.1 Jessica’s varying \( \hat{r} \) ............................... 100
  4.5.3.2 Ed’s blossoming understanding ......................... 103

4.6 Themes .............................................................. 103

5 CONCLUSIONS ......................................................... 112
  5.1 Rich reasoning for simple pendula ......................... 112
  5.2 Significant expansion of theory .............................. 113
  5.3 Looking outwards: connections and applications .......... 115

6 SUGGESTIONS FOR FUTURE WORK .............................. 117

REFERENCES ............................................................. 121

APPENDIX A –METHODS .................................................... 133

APPENDIX B –TRANSCRIPTS ............................................. 145

APPENDIX C –TRADITIONS AND THEORIES ....................... 204

APPENDIX D –RESOURCES NAMED .................................... 209

BIOGRAPHY OF THE AUTHOR ........................................ 211
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Five Traditions, as applied to PER</td>
<td>8</td>
</tr>
<tr>
<td>3.1</td>
<td>Plasticity: more plastic vs. more solid.</td>
<td>34</td>
</tr>
<tr>
<td>4.1</td>
<td>Subgraphs in <em>Coordinate Systems</em></td>
<td>57</td>
</tr>
<tr>
<td>4.2</td>
<td>Some resources in the locational subgraph</td>
<td>58</td>
</tr>
<tr>
<td>4.3</td>
<td>Some resources in the non-locational subgraph</td>
<td>58</td>
</tr>
<tr>
<td>4.4</td>
<td>Some pairs of resources within the <em>coordinate systems.</em></td>
<td>58</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 2.1 Resource graph for the motion of a tossed coin. ........................................... 25
Figure 3.1 A mass on a spring undergoing damped harmonic motion. .............................. 42
Figure 3.2 A comparison of the plasticity for forcesign and coordinate systems for Bill. ........ 49
Figure 4.1 The forces on a simple pendulum, with a physicist’s polar coordinate system shown. ................................................................. 62
Figure 4.2 A simple pendulum with two positions (left and right), two options for measuring \( \theta \), and all reasonable choices \( \hat{\theta} \). ........................................ 63
Figure 4.3 A simple pendulum with two positions (left and right), two options for measuring \( \theta \), and all reasonable choices for \( \hat{\mathbf{r}} \). ........................................ 64
Figure 4.4 A general resource graph showing the coordinate system choosing process between polar and Cartesian, with some details omitted. .................................................. 66
Figure 4.5 Two coordinate systems from Wes. ................................................................. 70
Figure 4.6 Derek and Wes use four definitions of \( \theta \) at different points in the two miniviews. ................................................................. 72
Figure 4.7 Derek’s resource graph from the week 4 miniview. ........................................... 78
Figure 4.8 Wes’s resource graph for the week 4 miniview. ............................................. 79
Figure 4.9 Two plasticity charts for the Week 4 miniview. ............................................. 81
Figure 4.10 A comparison of polar and Cartesian’s plasticity for Wes in weeks 4 and 10. .... 84
Figure 4.11 A comparison of the plasticity of Derek’s polar resource in weeks 4 and 10. .... 106
Figure 4.12 Derek’s resource graph for the week 10 miniview. ........................................ 107
Figure 4.13 Wes’s resource graph for the week 10 miniview. .......................................... 107
Figure 4.14 Resource graph for Rose’s choice of \( \hat{\theta} \). ............................................. 108
Figure 4.15 Resource graph for Rose’s choice of \( \hat{x} \). ............................................. 108
Figure 4.16 The students use a similar process to decide the direction and value for each coordinate as they do to choose between polar and Cartesian, but the discussion is more elaborate. ........................................ 109
Figure 4.17 Jessica uses two definitions for $\mathbf{r}$: $\mathbf{r}_1$ and $\mathbf{r}_2$ are similar to $\mathbf{y}$ in Cartesian coordinates.  

Figure 4.18 Two plasticity charts showing the relative plasticity of the polar resource for Wes and Derek and for Rose, Ed, and Jessica.
Chapter 1
INTRODUCTION

To better understand the processes of student learning, researchers build cognitive models that are consistent with observations of student behavior and can predict future behavior[1]. Many different theoretical approaches have been developed by education researchers that account for different aspects of student cognition and development[2, 3, 4, 5, 6, 7]. Physics Education Research (PER) is concerned with modeling, predicting, and optimizing student behavior and learning about physics, usually in formal teaching environments. As an interdisciplinary field, it relies heavily on physics content knowledge, educational theory, and cognitive science. A better understanding of learning processes and outcomes is integral to improving said learning. These two goals – understanding and teaching – are often commingled in a single research project in a specific content area. This thesis is concerned with the former goal of building understanding, and not the latter goal of improving teaching. However, just as commingled studies can use their understanding to improve teaching, the results of this thesis may be used to improve teaching.

The PER community has produced hundreds of papers about students’ conceptions of introductory physics, covering nearly every chapter in a standard textbook[8, 9, 10]. A multitude of organizational theories of cognition has been produced, ranging in size and scope from phenomenological primitives[2] to misconceptions[11], and many points in between[12, 13, 14]. Researchers have found that student epistemologies[2, 15, 16, 17, 18], content knowledge[19], and metacognitive skills[20, 21] are important.
Very little research has investigated upper-level student ideas in physics.\textsuperscript{1} Though introductory students have problems aplenty\textsuperscript{[30, 31, 32]}, and though many upper-level problems can be traced to lower-level difficulties\textsuperscript{[33]}, physics educators expect that many of these issues will be resolved as the student progresses through the program. As the supply of upper-level students, though small, seems unlikely to disappear, a worthy issue to investigate is how upper-level students think about topics in more depth than an introductory course can offer. In this thesis, I look at how intermediate mechanics students think about damped harmonic motion and how they think about polar coordinates in an undamped simple pendulum. The experimental work is used to illustrate the substantial theoretical work.

In Chapter 2, Theoretical Frameworks, I introduce metaphors that have guided education research, then outline five research traditions from Cognitive Science, Mathematics Education Research (MER), and PER. From them, I select one framework (Resource Theory) from one tradition (Pieces). Resource Theory is a general knowledge-in-pieces schema theory from physics education research. It bridges cognitive science and education research to describe the phenomenology of problem solving. Resources are small, reusable pieces of thought that make up concepts and arguments. The physical context and cognitive state of the user determine which resources are available to be activated; different people have different resources about different things. Over time, resources may develop, acquiring new meanings as they activate in different situations. In discussing Resources, I collect and develop a list of resources’ properties, both singly and in groups, as well as introduce heuristics for identifying resource use in interactions.

In Resource Theory, the generation of resources is not well understood. In Chapter 3, Plasticity, I introduce “plasticity,” a continuum for describing the development of resources. As a theory, it draws from both Resources and Process/Object. To

\textsuperscript{1}Notable exceptions include \textsuperscript{[22, 23, 24, 25, 26, 27, 28, 29]}. 

2
explore plasticity, I examine one student’s reasoning as he figures out the signs of forces when writing Newton’s Second Law for a damped oscillator. The student uses a coordinate systems resource to justify his reasoning about the direction of each force, and the plasticity of his forcesign resource changes over the course of the interaction.

The coordinate systems resource is not always easy for students to apply. In Chapter 4, Coordinate Systems, I select the coordinate systems resource for extensive examination through one touchstone problem: the simple pendulum. First, I detail the constituent resources in coordinate systems and introduce the simple pendulum and a common physicist coordinate system for solving for the position as a function of time. I then present students’ resource use in setting up coordinate systems for the simple pendulum. In the first set of interactions, I look at how two students’ resources’ plasticity changes from week 4 to week 10 in a semester, using small group interviews. In the second set, I look in depth at some of the resources that form the polar coordinate system for five similar students in a Homework Help Session (HHS).

Chapter 5 presents my conclusions, and Chapter 6 suggests some promising avenues for future work. In a theoretical thesis such as this, any experiment presented serves only as plausibility and existence proof. So as to minimize distraction, I collect the details of my experimental methods in Appendix A, Methods. Because the data are qualitative in nature, I include complete transcripts in Appendix B, Transcripts, for all of the episodes examined in depth. They do not constitute the whole of data acquired for this project. Appendix C, Traditions and Theories, discusses grain sizes of theories and research traditions, and contains some useful background material for a meta-theoretical discussion in Chapter 2, Theoretical Frameworks. Appendix D, Resources Named, is an index of resources named in this thesis.
Chapter 2
THEORETICAL FRAMEWORKS

PER is an interdisciplinary field which draws from cognitive and learning science, from physics content knowledge, and from other discipline-based educational research. Each of these fields holds its own research traditions, which determine the methods, research questions, and solutions available to researchers. In this chapter, I introduce five research traditions, compare them briefly as applied to PER, and select two for further development. Two of these traditions come from cognitive science (Connectionism and the Ecological Approach); one developed for mathematics education research (Process/Object); one from conventional education research (Misconceptions); and one grown from PER and cognitive science combined (Pieces). The former two share a tension about research methods and standards of proof; the latter two are commonly used in PER. All five are very different as theories of learning and interaction.

2.1 Metaphors we’ve learned by

Before launching into a discussion of modern learning theories, it is illustrative to examine some older paradigms – and their metaphors – for learning. Martindale[34] notes that different metaphors for the mind fall into (and out of) vogue as the dominant technology for the time changes.

The simplest idea is that the mind is like a dry jug. The role of the teacher is to impart truth unto empty-headed students, filling the jug with knowledge. This idea has simplistic appeal: teachers need not attend to prior student knowledge because it does not exist. However, as students often know some things when
entering our classes, a more productive metaphor might be that the mind is like a slate or whiteboard. The role of the teacher is then to fill the slate with correct information, erasing any incorrect former ideas and replacing them with appropriate truths. The improved metaphor allows teachers to improve or fix prior ideas, but it does not impart how teachers may “write” on students’ minds. Furthermore, it leaves students in an entirely passive frame, as silent recipients of instructor ideas.

A more active metaphor could be that the mind is like a single muscle. Through vigorous and repetitive exercise, it can grow in strength and acuity. Using this metaphor, students should study difficult subjects like physics and Latin to strengthen their minds. This metaphor is also responsible for the idea that students should repeatedly calculate sums, or copy out lines: just as a weightlifter does many reps with weights, a student’s mind must do many reps with common exercises to build competence. The weightlifter can use his strength in daily life as a dockworker, even though he gained it through lifting weights, not freight. Using mind-as-muscle, the content of students’ exercises matters less than the fact that they study difficult tasks: the mental strength garnered should be applicable even if the subject matter is not.

Unlike mind-as-jug, mind-as-muscle has continuing utility for research and learning. Repetition makes complicated muscle tasks (like walking) seem more simple; repetition can also make complicated mental tasks (like algebra) seem more simple. However, there are limits to the continued utility of repetition. If endless reps build muscle strength, then increased time on task should be the most important measure of building mental competence. However, in a study on student learning when using computer-based data acquisition tools, Redish et al.[35] found that one hour of targeted instruction on the concepts of velocity (as represented in velocity vs. time graphs) was more valuable than four hours of lectures on the topic. As it turns out, the content and method of instruction is more valuable than the quantity of it.
Mind-as-muscle has garnered new support in recent years as a result of dementia research in the elderly. It is the driving metaphor behind encouraging an aging population to do crossword puzzles or sudoku: research shows that keeping the mind active may delay the onset of dementia. However, other elder research shows that variety is also important: seniors should engage in many different tasks rather than immersing themselves exclusively in one. This variety in tasks could be likened to stretching a muscle, instead of simply strengthening it; however, the mind-as-muscle metaphor is not usually stated with this kind of flexibility.

A modern metaphor, often stated in reverse, is that the mind is like a computer. Under this model, eyes, ears, and other sensory organs perform analogous input functions to the brain as keyboards, mice, and other I/O devices do to a computer’s CPU. Short-term memory functions like RAM, and long-term storage functions like an infinite hard drive. On the surface, this metaphor seems useful in describing memory models congruent with psychological research. Short-term memory and RAM are both limited in capacity, but their contents are readily available for computation. In a computer, all data must pass through RAM before being written to the hard drive; in the brain, information passes through short-term memory before being stored in long-term storage. Similarly, humans and computers do not permanently record all of their calculations; only a pre-processed few are selected for storage. It is possible, but rare, for a computer to write all inputs to disk. It is impossible for a human to pay attention to all possible stimuli, let alone commit all details to long-term memory. The difficulty in recording inputs is compounded by the requisite interpretation in converting raw data into memory.

As computers have become more powerful, some of their developments have mirrored discoveries in human memory research. It was once thought that humans had one kind of short-term memory, but now it has been shown that auditory and visual short-term memory are somewhat separate. Similarly, computers used to
have a single processor and RAM bank for sound and graphic calculations; in the
1990s, having separate graphics and sound cards became commonplace.

In addition to obvious compositional differences (silicon-based vs. carbon-based;
hydrophobic vs. hydrophilic), the computer/brain metaphor has some other, more
subtle breakdowns. People tend to forget things easily and normally; computers
rarely lose data from their hard drives (absent hardware failure). Memory research
suggests that, absent physical trauma, memories cannot be erased (merely uncon-
nected); deleting files and writing over their former locations is commonplace in
computing. Further discussion of memory research is beyond the scope of this
metaphor.

All of these metaphors focus on the contents of the mind – what students know
– and, to a lesser extent, how that information enters the mind. However, the mind
is not an undifferentiated jumble of facts. Organization and connections between
ideas are vital for their appropriate recall and use. In the robust traditions that
follow, I focus on three core questions:

- How do individuals learn?
- How is knowledge organized?
- How is knowledge measured?

In choosing these three questions, I have chosen to focus on the role of a student
as a learner, not the professor as a teacher. I have also chosen to focus on indi-
vidual students, not social groups. In keeping with the rest of the PER field, the
students in question are children or adults. Even though I use classroom language –
“student”, “teacher” – it is understood that these processes occur outside of formal
environments as well.
<table>
<thead>
<tr>
<th>Tradition</th>
<th>Major strength</th>
<th>Major flaw</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical Cognitive Psych</td>
<td>Large N studies lend statistical power to conclusions.</td>
<td>Situations are divorced from natural context, lowering real-life applicability.</td>
</tr>
<tr>
<td>The mind is measurable</td>
<td></td>
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<tr>
<td>Ecological Approach</td>
<td>Descriptions of behavior have great depth.</td>
<td>Emphasis on natural settings limits control of variables.</td>
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<tr>
<td>Context determines learning</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Misconceptions</td>
<td>Commonly used in educational literature.</td>
<td>Descriptions focus on (in)correctness of student ideas.</td>
</tr>
<tr>
<td>Large-scale, robust ideas</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pieces</td>
<td>Descriptions of behavior have great depth.</td>
<td>Theory may be too general to make specific predictions.</td>
</tr>
<tr>
<td>Fine-scale local coherence</td>
<td></td>
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</tr>
<tr>
<td>Process/Object</td>
<td>Ideas develop over time.</td>
<td>Theory emphasizes formal learning environments.</td>
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<td>Procedures reify to concepts</td>
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### 2.2 Five traditions: an overview

In the following subsections, I present a brief introduction to five research traditions\(^1\) that address the three core questions and shape research in their respective fields. All five relate back to Constructivism, the idea that students construct knowledge from experiences and existing knowledge. Constructivism was first popularized to English and French speaking audiences by Piaget\([5]\). From such a simple premise comes remarkable diversity of detail.

After introducing these five shaping traditions, the following section examines the details of the theoretical framework of this thesis and its relations to the five shapers. Table 2.1 summarizes the five traditions and how they might relate to PER.

\(^1\)Appendix C, Traditions and Theories, discusses some of the philosophy of science relating to how different theoretical frameworks may interact.
2.2.1 Cognitive science

Cognitive psychology is the branch of psychology which deals with modeling the mind and its’ processes. In contrast, clinical psychology is concerned with the diagnosis and treatment of psychological distress or disorder. Operant psychology, another branch of psychology, is concerned with the description and replication of specific behaviors, and social psychology concerns itself with the development and interactions within and between groups. Within cognitive psychology, a number of traditions have developed around the general idea that the mind interprets information to make decisions, both consciously and unconsciously. Among the traditions are Information Processing, Connectionism, and Neural Networks. All three of these ally with computer science and artificial intelligence research. The overlap between cognitive psychology, computer science, artificial intelligence, and network models is called cognitive science.

Of the traditions within modern cognitive science, Information Processing is the oldest. Brought to the fore in Broadbent’s[36] seminal book, Information Processing uses the mind-as-brain metaphor to help explain how people think. Using that metaphor, the mind could be thought of as like a serial or parallel processor, and either centralized or distributed. Serial processors follow a single “train of thought”; parallel processors can work on multiple problems at once. A centralized processor has one decision-making unit; a distributed one spreads the load of decision-making across multiple units. In the 1980s, a model of the mind as a parallel distributed network arose[37]. This model helped birth the Connectionism tradition.

Connectionism is the tradition within cognitive psychology that posits that models of the mind should be reducible to brain activity (among other claims). This reductionism is attractive to physicists, who classically hold that physical behavior should be reducible (in principle) to a small number of physical mechanisms.
In neurology, groups of neurons fire together to produce different thoughts. Two different thoughts may involve two overlapping sets of neuronal firings. More complicated thoughts produce larger firing cascades, congruent with the idea that ideas may nest within other ideas. However, neurological studies have not produced a complete one-to-one connection between brain activity and cognitive processes. This lack of connection is not necessarily because the two fields conflict; rather, it speaks to a hole in scientific understanding. To help bridge that hole, it is useful to note that some of Connectionism’s strengths – in priming or object recognition, for example – have also been observed in studies of semantic priming.

An easy and common visualization for cognitive psychology is graphs[37, 38]. A graph is, in its simplest form, a picture denoting information symbolically and spatially. We can think of ideas and their connections, both temporary and permanent, as forming a ball-and-stick style graph. Ideas are symbolized by nodes and their connections by edges, commonly drawn as circles and arrows, respectively. Traversing a graph means to jump from node to node along excitatory connections, activating each node as it is visited. Just as the links between neurons are directional, the graphs used to model them use arrows between the nodes. They are called “digraphs,” a contraction of “directional graphs.” Using graphs, learning is represented by building new links, or strengthening old ones. People construct understanding as they build networks of linked ideas.

Using graph traversal, the process of identifying a printed word involves activating nodes for each of the letters, which then activate the node for the word proper. Identifying each letter as a specific letter involves both excitatory connections to the nodes that make up that letter and inhibitory connections to the ones which do not. For example, recognizing a rightward-pointing curve as a “C” requires that a curve node activates, but a central bar (“-”) one does not. If it did, a “G” might be recognized instead. Similarly, recognizing “CARGO” as a word involves exciting
links to C, A, R, G, O, their order, and a meaning. It involves inhibiting links to B and E, for example. (B and E separate CARGO from BARGE.) In Connectionism, researchers focus on recognizing sub-letter symbols. Using Information Processing, they might focus on word recognition. In both cases, the principle of graph traversal is the same. A rich and varied literature on symbol and word recognition and manipulation suggests that readers can omit some information in words and sentences, but that they might not ordinarily read at a letter- or sub-letter-level[39, 40, 41].

This trivial example of graph traversal may not seem directly relevant to physics learning, which is often more complicated. However, it serves a purpose. Letter and word recognition time can be clearly distinguished from the time needed to distinguish non-letters and word-like non-words: recognizing familiar words is faster than recognizing non-words. This experiment supports a cross-linked graph-like interpretation of cognition[37]. The excitatory and inhibitory nature of connections in a digraph have been observed in students’ reasoning about physics. For example, students may be able to draw a free-body diagram – a diagram showing all the forces acting on an object – given a verbal description, but unable to invent a verbal description given a free-body diagram[31]. I return to graphs and their emphasis on connections between ideas in Section 2.3, Resource Theory.

In addition to graph traversal, cognitive psychology uses graphs to illustrate priming. When a node activates, it sends signals to each of its connected nodes in accordance with the connection type: strong for strong, weak for weak, excitatory for excitatory, and inhibitory for inhibitory. These connections may cause other nodes to activate, thus spreading activation around a graph. Before these other nodes activate, but after some of the nodes that link to them have activated, they are primed. Priming is measurable; a primed node takes less to activate it than an unprimed node. Studies of semantic priming illustrate this effect[42]. If, for example, “bread” is recognized, then a related word, like “butter,” will be recognized more
quickly and accurately than a non-word like “murgle” or an unrelated word, like “equal.”

A word/non-word recognition study might use cognitive psychology to guide its experimental method, which commonly goes as follows:

*Find subjects* A large pool of subjects\(^2\) adept at identifying words would be identified and recruited. For a word identification study, the general adult populace is probably sufficient.

*Formulate hypotheses* Each hypothesis is measurable and stated in terms of a null hypothesis. For example, “The amount of time to identify words and non-words is the same for this population.”

*Present experiment* In controlled situations, present words and non-words. For example, subjects may press different buttons depending if a presented stimulus is a word or non-word. Data would record both the successful hit rate and the pause before selecting.

*Perform statistical analysis* Using appropriate statistics, accept or reject the null hypotheses.

It is notable that these studies draw much of their power from statistics and tightly controlled experiments. Because of the large numbers these studies often require, the ideas studied must be present in the general population. Should PER adopt this tradition, with its appealing reductionism and scientific bent, the small numbers of upper-division physics students and professional physicists will severely limit the statistical power or topics selected.

\(^2\)Education research uses “students”; cognitive research, “subjects” or “participants.” Here I adopt the language of cognitive science.
2.2.2 Ecological Approach

Cognitive psychology often concerns itself with what physicists[14] consider “zero-friction” experiments. A “zero-friction” experiment controls so many variables and divorces the subject matter from ordinary situations so much that the experiments are akin to physics laboratory experiments which suppose that friction does not exist and objects in motion will stay in motion forever. In psychology, a study in which participants memorize lists of words, then repeat them some time later is a zero-friction experiment. The results can be used to indicate the extent of short-term memory, or distinguish between two memorization strategies. However, the task is artificial and does not generalize easily to science learning situations.

As an applied field, physics education research is concerned with improving student reasoning in physics. To investigate how people actually learn and use science, PER turns to a different tradition: the Ecological Approach.

Unlike Connectionism, which stresses a mind-brain link, the Ecological Approach assumes that the culture and context of cognition determine the content. Physicists find the reductionism of the Connectionism attractive. As educators, however, they need the practicality of the Ecological Approach. The two traditions are not necessarily opposed, but their foci and thus their methods differ. The Ecological Approach stresses that the context of a problem is important to be able to solve it. Experiments with grocery shoppers, for example, find that people are good at buying the “best deals” quickly, a task which can involve significant mental arithmetic[43]. When asked to perform similar arithmetic tasks outside of a shopping context, they cannot. Similar research with Brazilian street vendors[44] examined child candy sellers on the streets of Brazil, finding that they performed significant mental arithmetic quickly and accurately, using algorithms not taught in schools. The field of Ethnomathematics embraces and studies the context-dependence of mathematical
thought[45]. Using the Ecological Approach, learning is described as a process of constructing new understanding in response to new contexts. To properly study the learning, a full description of the context is necessary.

In physics education research, the extreme context-dependence of students’ reasoning about physics is well-documented[2, 17, 20, 46, 47]. Novices classify problems based on surface features (“ramp problems” or “pulley problems”), while experts classify on solution methods (“energy problems” or “forces problems”)[48]. Providing a context-rich problem[49] or an atypical problem[50] in physics can confound these efforts and significantly increase the difficulty of the problem.

This emphasis on context means that studies following the Ecological Approach cannot recruit large numbers into their laboratories the way Cognitive Science studies can, nor is it possible for them to use bulky or intrusive equipment in the same manner. A study of eating habits and stress using the Ecological Approach might be conducted in the following manner:

**Identify a topic** Find an activity, such as eating, that people engage in normally, and identify a group of people to study, such as students nearing their exams.

**Formulate Hypotheses (optional)** Each hypothesis is measurable and stated in terms of a null hypothesis. For example, “When faced with increased workload, subjects’ eating patterns do not change.”

**Determine behavior** Observe subjects as they perform that activity, or ask them to report their actions (such as in a food diary or in interviews). These probes are designed to be minimally invasive so that subjects do not change their behavior in response to the probe.

**Find trends** Reduce the data, possibly through quantification, and, if possible, perform statistical analyses.

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3Some studies using the Ecological Approach do not seek to formulate or test hypotheses, opting instead to provide a full and rich description of behavior.
In this process, subjects are not subjected to controlled conditions in the laboratory to study the effects of increased stress; instead, their responses are determined in situ. A second major difference between this process and the preceding is in the types of statistical analyses possible to perform. In the preceding process, the data is inherently numerical, continuous, and groups can be randomly assigned. In this process, the data has been quantified, and groups are not randomly assigned; the number of participants may also be smaller. These differences make different statistical tests appropriate.\footnote{The details of these statistical differences are outside the realm of this qualitative study.}

In addition to these technical differences, studies using the Ecological Approach may also use qualitative methods, like interviews, in determining behavior. Data can often take the form of video or audio recordings, field notes, or open-end questionnaires.

### 2.2.3 Conceptions

Both Cognitive psychology and the Ecological Approach are grounded in cognitive science, but they are not the only traditions within their field. Educational psychology is a related field with related traditions. In PER, two Traditions have shaped research and curricula: Conceptions and Pieces Theory. Elby[51] termed the Conceptions Tradition “misconceptions constructivism” and Pieces “fine-grained constructivism.” Pieces is discussed in the next subsection.

Conceptions are large-scale, robust, and stable ideas about a given topic[52] (and references therein). For example, the medieval impetus concept of motion requires that a motive force imbues an object with the ability to move[53]. Once that force is used up, the object naturally stops. Someone holding this conception would say that a book sliding across a table naturally stops because the force needed to move it is used up. Alternately, a rock swung in a circle will continue to move in a curve
after release because it has a curvilinear impetus. As that curvilinear impetus is used up, the path will straighten.

Often, these conceptions are wrong and need to be replaced with approved scientific ones. The impetus concept of motion is at odds with the approved Newtonian concept of motion, which holds that an object in motion stays in motion until acted on by an outside force. Forces are thus interactions between two objects, and not something that can be used up. Both of these conceptions are monolithic: they must be accepted or rejected whole, and to accept one of them is to reject the other. Learning is a process of building and changing conceptions through experience[54].

On occasion, researchers find that students do not wholly subscribe to one conception, but possess a hybrid model[55, 56], two competing conceptions[57], or the students may be in transition between two models[58].

Conceptions, as a tradition, focuses on the correctness of student ideas: an idea is either correct – scientific – or it represents a misconception. To soften the language and admit that students are reasonable (if wrong), “alternative conceptions” has been proposed in place of “misconceptions”[59]. To instill hope that these wrong ideas will be corrected, “preconception” is sometimes used[60]. These comparisons between student ideas and correct ideas encourage researchers using Conceptions to fix students’ wrong ideas. Thus, questions of teaching and learning are more central to Conceptions than they are to the Ecological Approach or Cognitive science.

A classic research method in PER employs Conceptions and was pioneered by the University of Washington Physics Education Group (UW-PEG). Generally speaking, the method is as follows[61, 62, 63, 64, 27]

*Identify physics* Choose an important topic in physics.
**Identify misconceptions** Using individual interviews, possibly involving equipment ("demonstration interview") or instruction ("teaching interview"), identify the misconceptions that students hold.

**Determine prevalence** Generalize interview data into a short series of questions, which may be open-ended or multiple choice, and administer the distilled questions to hundreds of students. Score the responses to determine which (mis)conceptions are prevalent in the population.

**Fix misconceptions** Write or edit curricula[65, 66] to elicit misconceptions, confront them with evidence, and encourage students to resolve any discrepancy in favor of correct conceptions. Administer the same or similar series of questions to determine the new distribution of (mis)conceptions.

**Repeat** Repeat the prior three steps until the curriculum produces the same prevalence of correct answers in the target population (after instruction) as in the instructors (before instruction). Traditionally, curricula from UW-PEG effects a change in right answer frequency from \(\approx20\%\) to \(\approx80\%\); with gains like that, sophisticated statistical analyses are usually omitted.

It is notable that early papers from the UW-PEG explicitly cited Piagetian constructivism and misconceptions as their guiding principles[61], while later papers do not mention an explicit theoretical basis[67]. Scherr[52] contends that later studies still used the Conceptions tradition.

In response to evidence that students do not necessarily hold one monolith to the exclusion of others, two different foci have developed: a focus on the context of a problem, and a focus on the grain size of a conception. However, Conceptions research retains its emphasis on the correctness of student ideas[51, 52].

By considering context, we admit that students may respond differently to questions, though they be identical to the asker, seem different to the askee. By consider-
ing grain size, we admit that a student need not be universally consistent: students may have conflicting ideas in different domains. By limiting the size and scope of a conception, we enter the domain of Pieces.

2.2.4 Pieces

Pieces is a fine-grained constructivist schema theory. As a theory, it differs from Conceptions in two major ways. Both of those theoretical differences stem from a single point of departure, and they lead to different implications for research and teaching.

Pieces proposes that student ideas might vary easily in different situations and at different times. The myriad combinations of smaller thoughts make up the complex behavior we witness in students. Just as computer programmers reuse modular portions of their code in new projects, students reuse previous ideas in new situations, a connection explicitly exploited for Pieces’ early development[68, 69, 70]. For example, a computer programmer needing to sort a list of words writes a sorting algorithm. Later, when performing a mail merge, he does not need to rewrite the sorting algorithm; he merely reuses it in a new context. Similarly, a student may use the idea that more cause means more effect in explaining both the destruction of vehicles in high speed (vs. low speed) collisions and reuse it in explaining the brightness of a light bulb attached to two (as opposed to one) batteries. This point of departure – that variance is normal and expected – has a several interesting consequences.

In contrast to Conceptions’ assumption that students hold a single (or perhaps two) robust and monolithic ideas on a subject, Pieces claims that students apply a number of “mini-generalizations”[51] depending on the student’s mental state, the context of the question, and the situation at hand. This insistence on the context-
dependence of student thought harkens to the Ecological approach and suggests that qualitative research methods are appropriate.

These mini-generalizations might form a coherent network – in which case they might appear like a conception – but the theory does not require them to do so[52]. In this tradition, learning can be seen as strengthening old links or making new ones. In dropping to a smaller grain size and focusing on connections between ideas, Pieces draws from Connectionist models.

Conceptions focuses on the correctness of student ideas; Pieces focuses on the connections between them. In dropping the emphasis on veracity, Pieces is more able to find partial value in students ideas[51]. The resulting curricula invite students to “refine their intuitions” [71] instead of eliciting, confronting, and resolving their misconceptions[66]. A more fruitful theory of student behavior is important to a field whose primary goal is to model and optimize that behavior.

A partial list of the theories within Pieces Theory includes resources[70], facets[72], intuitive rules[73], factual units[74], and phenomenological primitives (“p-prims”) [2]. Each of these theories has particular strengths and weaknesses for use in analyzing some kinds of data or student behavior, but they do not form a linearly independent or complete set of theories for cognition. In Section 2.3, I return to Resource Theory and go into much more detail about the various elements introduced in this section.

2.2.5 Process/Object

In Mathematics Education Research (MER), one research Tradition is Process/Object. Like Pieces, Process/Object comes from a larger Tradition of Schema Theories. Pieces focuses on the connections between student ideas; Process/Object focuses on how chains of ideas become automatic. Process/Object presents two perspectives on student thought, process and object[4]. A process is recipe-like series
of operations for calculating some mathematical idea. Objects are the ingredients in processes; processes act on objects.

Consider the idea of mathematical function. Using a process perspective, functions denote a series of operations for converting values of one variable into values of the other: the linear function denoted \( y = 3x - 7 \) takes an x-value as an input and produces a y-value as an output. In contrast, an object view of the same function would see the same mathematical statement as a set of \((x, y)\) pairs. This set might be represented as a line on a Cartesian graph. The process view of function – that functions are a series of operations – is very similar to the definition of “operational definition”[75], though the two ideas come from different fields.

This process view of function depends on an object view of variable. The object view of function might be used in a later process, such as taking the derivative. Generally speaking, a student must posses the process perspective for an idea before reifying it to the object perspective. A process view of function precedes an object view, and the object view of function is a necessary prerequisite for a process view of derivative. Thus, learning is modeled as the reification of processes into objects.

As posed by Sfard, processes and objects are two sides of the same coin. Once developed, both sides are available for the user to choose between as a problem warrants. Some problems prefer a process perspective, and others prefer an object perspective. If a student only possesses the process perspective, then some problems may be impossible without lengthy calculation. Given a list of values for x and a third degree polynomial equation,

\[
y = x^3 + 6x^2 - 2x
\]  
(2.1)

a student with a process view of function can calculate the corresponding y-values. If the student is then given a similar function,

\[
y = x^3 + 6x^2 - 2x + 4
\]  
(2.2)
he will again be able to calculate the new y-values using the same procedure. With an object perspective, however, extended calculation is unnecessary: the second expression for y is simply 4 more than the first. An object perspective might “see” the second equation as moving upwards on a graph, but retaining the same shape. In PER, curriculum in the context of waves has been developed to encourage and refine this object view of function in the context of graphs[76].

Process/Object is a reification theory. As such, it describes how people, usually students, become more facile and flexible with their use of specific ideas, usually mathematics ideas. Process/Object has been commonly applied to the mathematical concepts of function[1], limit[77], and differential[78].

Process/Object has several cousins within MER. APOS (“action, process, object, schema”) [79], adds “action,” a state where the process is not yet internalized, and “schema,” a state where the object is so tightly reified that it becomes a schema in its own right. A PER counterpart is von Aufschnaiter’s levels of complexity[80]. A similar – though subtly different – theory is Tall’s procepts[81]: “procept” is a contraction of “process” and “concept.” The RBC model for abstraction[82, 83] is another cousin which proposes three epistemic actions (“Recognizing,” “Building-with,” and “Constructing”) inferable from behavior. I detail the RBC model and marry it to Resource Theory in Section 3.1.2.

MER has developed as a field independent of PER. Even though mathematics and physics are related subjects in school, the two discipline-based education research fields are largely separate. Sherin’s work[84] on mathematical forms, Meredith and Marrongelle’s work[85] on the physics/calculus link, and Tuminaro’s work[86] on epistemic games are notable exceptions. In this thesis, I synthesize Process/Object ideas with Pieces ideas to show how Pieces may be connected with data (Section 2.3.2, Resource Heuristics, and Section 3.2, Plasticity Heuristics) and how they develop with time (Section 3.1, Plasticity).
2.3 Resource Theory

Resource Theory is a constructivist schema theory which bridges neuro-cognitive models of the brain and results from education research[87] to describe the phenomenology of problem solving[2]. Resources are small, reusable pieces of thought and therefore Resource Theory has its roots in the Pieces tradition. Resource Theory focuses on the connections between different ideas in physics, and therefore it draws from Cognitive Science. I choose Resource Theory as my central theoretical framework in this thesis, but also draw from theories in Process/Object, Conceptions, and Cognitive Science.

Because of Resource Theory’s genesis in the Pieces tradition, most examples of resources in the literature primarily focus on primitives[2]. Some examples include “effect dies away,” which describes the motion of a box sliding on a floor, the ringing of a struck bell, a person’s motivation, and other phenomena. A mathematical equivalent exists in symbolic forms[88]. Though most described resources are primitive and thought of as having no internal structure, I describe a larger resource, coordinate systems, with much internal structure in Chapter 4.

As originally published[70], resources were intentionally vaguely defined. Later papers elaborate on the theory and make more explicit connections between resources and other theories[17, 89, 21, 14, 87, 25, 86, 90, 91]. As Resource Theory has developed, different aspects of student cognition have been found important, including epistemology[17], metacognition[21], physics and mathematics content knowledge[86], and problem solving skills[25]. Representations of linked resources have been described[90] and made consistent with the model of coordination

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5Forms are general (mathematical) elements of a structure, which students apply to specific situations.
classes[92]. Many aspects have not been explored, including their self-efficacy\(^6\) and literacy.

As a theory, Resources has been used in conjunction with epistemic games and frames[93], students' sense of physical mechanism[94], and student reasoning in nearly-novel situations[25]. It has been used to support curriculum development in areas of introductory algebra-based physics[71], intuitive quantum physics[95], and intermediate classical mechanics[22, 96].

### 2.3.1 Resources, the units of Resource Theory

The fundamental unit of Resource Theory is the resource. Resources are small, reusable elements of thought[70]. Resources take their name from the computing metaphor: a computer uses resources to perform its computations, though the nature of those resources be vague. Based on the literature and my own work, I summarize the properties of individual resources and groups of resources in the sections below.

#### 2.3.1.1 Kinds of resources

Metaphorically speaking, an individual resource can be thought of as a shipping box[38]. Inside the box could be a factoid or bit of content knowledge[70, 90]. Or, the box may contain a reasoning primitive\(^7\) like a p-prim[2]. It may contain a belief about the nature of knowledge, of science, or of self[16, 97, 17]. It may contain a prompt to reexamine which resources are active[21]. Unlike the concepts and misconceptions in the Conceptions model, resources are not necessarily inherently correct or incorrect[2, 70].

\(^6\) (except inasmuch as it might affect their epistemological resource activations)
\(^7\) I use “primitive” in the sense of “irreducible,” not “backwater” or “savage.” No value judgement is implied.
Resources are individually nameable, such as coordinate systems or knowledge-from-authority[17]. Researchers name resources; students need not be aware of the resources used or their names.

In keeping with Pieces’ and cognitive science’s emphasis on individuals’ cognition, individual students hold resources. Resources are not socially negotiated, unlike social norms. Though individual resources are held by individuals, their expression and use in a specific context may be socially negotiated. However, I wish to draw a distinction between resources (held individually) and their appropriate expression (which may be socially determined). Different people may each hold a version of a given resource, and the construction and details of use may differ among people.

2.3.1.2 Two states

Resources have two states: active, and inactive. An active resource is being used; an inactive one is dormant. This two-state system harkens to neurology, in which neurons have two states. Because neurons also have a third state, “primed,” some researchers posit that resources may have a third state as well. This two-state property of resources is in accordance with Connectionism.

The physical context and cognitive state of the user determine which resources are available to be activated; not all people activate the same resources in response to the same prompts. The activation of resources occurs when their invocation, express or implicit, is used to support or form an argument.

Asking a child at the park, “Where’s the ball?” may activate resources for an activity (looking) as well as resources for balls as objects that are round and bouncy. The same question posed at bedtime may activate resources for the storyline activity as well as princess or dancing resources.
2.3.1.3 Connection schemes

In the ball question, multiple resources activate. These activations are not unconnected. Resources link with each other. These connections are commonly conceptualized as ball-and-stick style graphs[90]. Each resource is a node in the graph, and each connection is a directional link[38]. This network model is consistent with a model of coordination classes[92].

As an example, consider the motion of a tossed coin[90]. Figure 2.1 shows a possible resource graph. In considering this question, you probably activated a velocity resource. Activating velocity may have activated actuating agency and then forces as you consider why the coin moves. The motion dies away as the coin nears the top and slows down. Of course, to think about the coin in this manner, you’ve already activated object (instead of money), and you’re probably using part-for-whole to think about the center of mass motion instead of spinning or flipping.
Just as neuronal links may be excitatory or inhibitory, links between resources may promote or demote activation[87]. If the network tends to have the same structure repeatedly, then it is “stable.” If not, then it has been built “on the fly.”[70]. Stable networks consist of resources that are cognitively “nearby” each other: they tend to activate together[38]. 8

In this thesis, I pay special attention to the stability and strength of different link structures. In Chapter 3, Plasticity, I describe a simple linking network which includes a non-robust resource (forcesign), and I show how the network of resources stays the same as it becomes more solid in use. In Chapter 4, Coordinate Systems, I discuss how linking among resources within coordinate systems varies from person to person, and what the implications for the robustness of coordinate systems and some of its components may be.

2.3.1.4 Internal structure

In the ball question, one resource that could activate is princess. Princess is not a primitive idea: it has constituent ideas for what princesses are, their roles in fairytales, appropriate princess dress, etc. As a resource, princess illustrates that resources are nestable:[38] they may have internal linked structure made up of other resources. In the resource graph for the tossed coin (Figure 2.1) forces is certainly not primitive, but it links to other resources (like actuating agency) which are. This property of resources becomes extremely important in Chapter 4, Coordinate

8In terms of a multiply-contexted resource, if a resource activates in two contexts, those contexts may or may not be near each other. Consider the conservation of stuff resource. In quantum mechanics, the energy of a particle is conserved as it passes through a barrier, though students may think the energy decreases[91]. In a classic Piagetian experiment, water is poured from a short fat glass to a tall skinny one, but its volume remains the same, though children may think the volume increases[98]. In both cases, Conservation of Stuff is appropriately activated, but the cases bear no other resemblance. Thus, conservation of stuff could be nearby many otherwise disparate resources. Yet, if those other resources are disparate, they should not be near each other. To solve the addressing difficulty, resources need to be in two “places” at the same time. Obviously, ordinary buildings cannot have multiple locations. It may help to think of Starbucks, whose many locations are indistinguishable on the inside yet located on a variety of street corners.
Systems, where I explore the structure of one resource, *coordinate systems*, in the context of simple pendula.

I wish to draw a distinction between resources whose internal structure is accessible to the user, and resources whose internal structure is not. If their internal structure is explorable (but currently not explored) by the user, they may be called concepts. If their internal structure is no longer explorable, they may be called primitives. A large body of literature has identified both concepts and some kinds of primitives[2]. Because of the nestable nature of resources and the way in which resources can span many grain sizes of analysis, the resources discussed in this thesis will include examples of both. An index of all resources named in this thesis is included in Appendix D, Resources Named.

### 2.3.1.5 Notation conventions

In this thesis, a named resource, such as *coordinate systems*, is italicized in the text. On occasion, the tense or form may change for grammar reasons. For example, “Rose chooses a coordinate system where…”; “Rose’s choice of coordinate system…” In figures, a resource is shown as a circle. The sizes of circles have been chosen only with regards to the space on a page and the readability of text within them; no link is intended between circle size and resource size or importance.

### 2.3.2 Resource Heuristics

Though Resource Theory has been available and developed for some time, a collection of heuristics for identifying resources *in situ* has never been formally presented. In his seminal 1993 paper, diSessa[2] presented a list of 17 p-prim heuristics. However, resources and p-prims bear enough differences that a different list of heuristics is desirable. A short list of heuristics for resources follows:
\textbf{R1.} Resources are reusable. For an idea to be considered a resource, it must have sufficient duration to be reused. Thus one appearance of an idea is suggestive, but insufficient, to term that idea a resource.\textsuperscript{9}

\textbf{R2.} Resources may be referred to without exploring internal structure. It may be that the internal structure is unavailable, such as the case of primitives, or it may be that it is simply not currently explored by the user, such as the case of concepts. In either case, the referring need not be explicit: resources may activate without being explicitly called.

\textbf{R3.} Resources are nameable by researchers. A resource is a discrete bit of thought. While it can be alluring at times to refer to “resources relating to a coin toss”, for example, unspecific language of this nature does not identify the specific resources in question. This amount of detail is not always desirable, but it is possible in principle. Note that users need not name – or even be explicitly aware of – all the resources that they use.

\textbf{R4.} Resources activate. If a resource is not apparent in a given situation, that failure is not necessarily indicative that the resource does not exist, merely that it did not activate. One (or more) of the following conditions may be preventing its activation: another resource may be blocking it; the context may not be linked to it; it may not exist. To prove a lack is difficult. Only over many situations, in varied contexts, is it safe to say that a resource which has never activated does not exist.\textsuperscript{10}

\textsuperscript{9}The purpose of these heuristics is not to make a list of possible resources, but to understand the resources in use by a single student. However, in situations where many students make the same statement with only slight variations, one may posit that a resource common to all may be in use even if that resource might be quite plastic[20].

\textsuperscript{10}It is a somewhat philosophical question to wonder whether a resource exists which has never activated. Because resources are defined as activatable, if it could activate (and merely hasn’t), then it might exist.
These heuristics can be used to find “classical” resources: resources that are generally well-developed and used in many settings. What happens when students use resources that are not well-developed? How might those resources be identified? In the next Chapter, Plasticity, I present a continuum which details how resources may develop as well as heuristics for identifying less well-developed resources.

2.3.3 Resource Theory strengths

Resource Theory has several strengths to recommend it as a governing theory in PER.

PER is concerned with physics learning at all levels[99, 100, 50, 66]. Resources describes reasoning done by both experts and novices, as well as people somewhere in between[86], and so it can be applied to all the populations of interest to PER.

In a field as varied as PER, a wide variety of curricula have been developed to tackle a wide variety of topics in physics at different instructional levels. Resources can describe learning in many different curricula[90, 52], even when the curricula were not designed with Resources in mind.

As a theory, Resource Theory is well linked to may other theories of learning and social interaction[14]. It builds on a long tradition of small-scale models of learning[69, 72, 2, 92] while generalizing and expanding those theories to more accurately reflect learning in physics. It also draws from complementary theories in MER[4, 82, 83] and education research[59] to increase its descriptive power.

2.3.4 Open questions in Resource Theory

Traditions are more than just theories[101]. A Tradition in PER must include both theoretical and experimental methods for describing student behavior. Despite the number of studies which have used resources as inspiration or as an organizing frame, a complete operational definition of a resource or its activation has been
elusive. There is a disconnect between the methods of qualitative research commonly employed in PER, and the theories used to make sense of the data.

Resource Theory may be too vague. Because resources are general and by allowing resources to nest within each other, researchers in using Resource Theory have great flexibility when choosing a grain size at which to work. However, this flexibility sacrifices some specificity. What is – or is not – a resource? Resource heuristics R1 and R3 may help with this ambiguity.

Resource Theory may be too complex to model student reasoning in real world situations. An exhaustive list of which resources are active in a situation is neither desirable nor possible. Querying changes activations, resources come in different grain sizes, and there’s no obvious utility to having a list. That said, chronicling a partial list of activations has merit. In this thesis, it is understood that when I list active resources in a situation, there are resources active to which I do not attend.

The processes by which resources develop are not detailed or understood, other than to generally say that resources are constructed. Similarly, their genesis is not understood. In the next Chapter, Plasticity, I tackle questions of resource development without addressing the issue of resource genesis.
Chapter 3

PLASTICITY

What we learn for the sake of knowing, we hold; what we learn for the sake of accomplishing some ulterior end, we forget as soon as that end has been gained. This, too, is automatic action in the constitution of the mind itself, and it is fortunate and merciful that it is so, for otherwise our minds would be soon only rubbish-rooms.[102]

Suppose that you are studying a foreign language and there is a test coming up. You faithfully and intensively set about learning all you can about verb conjugation and noun genders. On the test, you do quite well, but find that you cannot recall rules for past perfect progressive conjugation afterwards.¹ What happened?

As you studied, you built resources for verb conjugation and vocabulary. During the exam, those resources linked properly with each other, as well as exam-taking and writing resources. Afterwards, it seems as if your past perfect progressive resource has melted away. Perhaps it was not as solid as you thought.

In Chapter 2, Theoretical Frameworks, I detailed the properties of resources and connected Resource Theory to a wide variety of theoretical frameworks. Chapter 2 closes with a discussion of open questions in Resource Theory. One such open question is the development of resources. In this chapter, I introduce “plasticity,” a continuum to help answer that question. I present some heuristics for identifying resources’ plasticity. To illustrate how plasticity can usefully describe students’ reasoning about physics, I give an extended example in which a student taking an intermediate mechanics class reasons about the forces in damped harmonic motion for a homework problem. I use the heuristics to reason about the plasticity of two

¹The past progressive tense indicates ongoing actions that happened in the past. “You were learning”; “he was eating.” The past perfect progressive indicates that these ongoing actions have ceased: “You had been learning”; “he had been eating.”
resources, *forcesign* and *coordinate systems*, and to reason about how that plasticity changes over the course of an approximately 6-minute episode.

### 3.1 Introducing Plasticity

One of the holes in Resource Theory is that it doesn’t describe how ideas become resources, or how resources could develop in time. To help fill this hole, I introduce *plasticity*. Plasticity is a continuum which extends Resource Theory to describe the genesis and development of resources. To do so, it draws on Process/Object’s emphasis on reification and on one of Process/Object’s cousins, the RBC model for abstraction.

Recall that in Process/Object theory, mathematical reasoning takes the form of performing processes on objects. For example, a process view of *function* sees $y = 2x + 7$ as a recipe for converting $x$-values into $y$-values. The process view of *function* depends on an object view of *variable*. An object view of *function* might be necessary to think productively about derivatives[77]. These ideas can be applied to resources. Some resources may be very process-like, requiring attention to the individual steps. They may be weakly connected to other resources. Other resources may be object-like, and appear as one well-developed ‘chunk’ used together with many other resources. The former can be thought of as “plastic,” the latter as “solid.” Because of the nested nature of resources, it is possible that plastic resources may contain within them more solid resources, just as processes are performed on objects.

In choosing the language “plastic” and “solid,” I hope to evoke physical sense of objects. A plastic object, like a lump of clay, deforms easily. The metaphor continues: as concrete dries, it becomes more resistant to deformation, and thus
more solid. Solid gravel is a useful component in wet concrete; solid resources often make up more plastic ones.

In neuroscience, “plasticity” can refer to a neural network’s ability to form new connections. Younger brains are more receptive to learning, and thus more plastic; older brains have been myelinated to increase processing speed at a cost of less learning power. This speed increase is also seen with solid resources: students reason more quickly with solid resources than they do with more plastic ones. I do not address the issue of learning power (which is not adequately defined in Resources) in this thesis.

3.1.1 Comparing plastic and solid

Having motivated a need for plasticity and evoked a sense of what plasticity might entail, I now turn to a more formal definition of the plasticity continuum. The two directions in the continuum are more solid and more plastic. A solid resource can be considered durable: its connections to other resources are plentiful, and its internal structure is unlikely to change under typical use. It has been used in many settings and is therefore connected to many other resources. If sufficiently complex, it might be called a concept in the education research literature[60, 103, 57, 58, 59]. Existing literature details solid resources and some of their properties[69, 104, 105, 70, 16, 17, 89, 106, 84, 107, 90]. Common student difficulties[108, 27, 109, 110, 61, 111] often can be reinterpreted[51, 52] as solid resource use in inappropriate situations.

Plastic resources, in contrast, are less durable in time or less stable in structure. While a solid resource may remain unchanged for years, a plastic one may only last “until the exam,” and an extremely plastic one may only last a class period. The more plastic a resource is, the less likely the user is able to apply it to new situations, and more explanation is needed to justify and explain its use. In terms of graphs,
<table>
<thead>
<tr>
<th></th>
<th>More plastic</th>
<th>More solid</th>
</tr>
</thead>
<tbody>
<tr>
<td>proto-resources</td>
<td></td>
<td>objects</td>
</tr>
<tr>
<td>last as little as</td>
<td></td>
<td>practically forever</td>
</tr>
<tr>
<td>a class period</td>
<td></td>
<td></td>
</tr>
<tr>
<td>few connections</td>
<td></td>
<td>many connections</td>
</tr>
<tr>
<td>to other resources</td>
<td></td>
<td></td>
</tr>
<tr>
<td>weak user</td>
<td></td>
<td>strong user committment</td>
</tr>
<tr>
<td>commitment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>many, explicit</td>
<td></td>
<td>no longer checked in</td>
</tr>
<tr>
<td>consistency checks</td>
<td></td>
<td>typical use</td>
</tr>
</tbody>
</table>

A plastic resource may only have a few connections to other resources, and those connections may be tenous or weak. The more solid a resource is, the more likely the user is to refer to the resource in diverse contexts without explaining its internal structure.

Because solid resources are so long-lived and well-connected, their use has solved many problems in the past for users. Therefore, users tend to be committed (attached) to their existence. In contrast, plastic resources are not surrounded by a rich network of strong connections, and users are not as committed to using – or keeping – them.

As those plastic resources develop connections, the user performs consistency checks to help find them a place in the network. In contrast, solid resources are rarely checked for consistency under typical use any more. Atypical use, such as when a user is confronted with confounding evidence, might prompt a consistency check as the user is forced to resolve the discrepancies. A rich literature on conceptual change theories[112, 92, 113, 114, 54, 115, 90] details some possible forms and methods of conceptual change.

The plasticity of a resource is independent of its veracity, in keeping with resources’ truth indeterminacy. Table 3.1 chronicles some of the properties of more plastic and more solid resources. Though the table has two columns, readers are
reminded that plasticity is a continuum, and that most resources will fall somewhere in the middle.

3.1.2 The RBC model for abstraction

To inform and improve the plasticity continuum, I draw from the RBC model for abstraction[83, 82], noting that it was originally intended to describe the reification and abstraction of mathematical ideas. The RBC model proposes three epistemic actions through which abstraction occurs and which may be inferred from behavior. These three actions - recognizing, building-with, and constructing - are dynamically nested. Recognizing, the simplest action of the three, occurs when a student realizes that a “familiar mathematical notion, process, or idea ... is inherent in a given mathematical situation.”[82] In Resource Theory, these recognized cognitive objects are resources. Recognition is thus synonymous with activation. Content resources such as these need not be restricted to mathematics; physics is another appropriate subject area. The specifics of which resources are recognized gives insight into students’ thought structure. A solid resource, being well-connected to many other resources, will be quickly activated. Ease of recognition is therefore a marker of solidity.

Once a familiar idea has been recognized, a student may build-with that idea to solve a local goal, such as solving a problem or justifying a statement. Several resources may need to be recognized and built-with at once. Under Resource Theory, activated resources form a web or graph that may be built on the fly. Thus, recognizing is akin to activation, and building-with is akin to building graphs on the fly. Because building-with and recognizing are two separate actions, the RBC model can describe behavior when students mention an idea, but don’t appear to know what to do with it. Instructors have all experienced moments where students can
name an idea relevant to a situation, but don’t know what to do with it; the RBC model formalizes this phenomena differently than social interaction models do[20].

In contrast to building-with, constructing has purpose and duration beyond solving a local goal. Constructing creates a less local, more abstract entity. As a construction becomes more durable, it becomes more consolidated and is no longer necessarily built on the fly. It becomes a resource in its own right, and therefore can be recognized or built-with in later local goals. The new-formed resource may be quite plastic, but as further constructions are added to it and as it compiles further, it can become more solid. Thus, constructing is a mechanism for increasing the solidity of specific resources as well as a mechanism for their generation. Extremely solid resources – rigid resources – have been so tightly compiled that their internal structure is not readily accessible to the user. A resource whose internal structure is no longer user-accessible is a primitive.\(^2\)

The process of abstracting and consolidating resources can be of long duration. Later in this chapter, I focus on an example of recognizing and building-with which shows different levels of plasticity in one encounter. In Chapter 4, Coordinate Systems, I explore a different example to show how the plasticity of a resource may change over the course of a semester, and how the plasticity of a resource may be related to the plasticity of resources within it.

### 3.2 Plasticity heuristics

The goal of much teaching is to help students develop solid resources that can be used quickly and easily in many contexts. In the terminology of Resource Theory, it is important for teachers to recognize where student resources are plastic and

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\(^2\)Consider the p-prim actuating agency, whose existence is predicated on a rudimentary understanding of cause and effect. Small babies do not seem to exhibit this p-prim, yet it is primitive to adults.
facilitate student development of more solid resources. Of course, not all resources need be solid.

For researchers using a theory, it is important to able to apply the theory to experimental situations. Using the RBC model, I can extend Resource Heuristics R1-R4 (Section 2.3.2) to find evidence of resources’ plasticity, enabling both teachers and researchers to apply plasticity to student language.

P1. Ease of use. The more solid a resource is, the more easily it can be recognized or built-with. This ease of use is directly related to the number and strength of connections that a resource has. Well-connected resources are more likely to activate in a variety of situations.

P2. Recency of construction. Often, but not always, the more recently a resource was constructed, the more plastic it will be. Counter-examples include infrequently used resources, which may be old but plastic (like a physics professor’s criterion for critically damped harmonic motion), or recently constructed “flashbulb” resources, which are so vivid that, despite their newness, are etched solidly upon the mind.³

P3. Need elaboration to evaluate. Users need to explicitly test plastic resources against other (often more solid) resources to determine if the plastic ones should be used in a given context. These tests often take the form of elaborative sense-making. In contrast, solid resources can be apprehended whole and are often quickly recognized without elaboration.

P4. Justification. Because plastic resources often are tested against solid ones, solid resources can justify the use of plastic ones. The degree to which a resource justifies another can be used to see how nearby[25] the two resources are.

³“Flashbulb” resources are like flashbulb memories.[116]
P5. Need rejustification or rederivation for extended use. In a long episode, as
resources fall out of working memory, very plastic resources may need to be
rebuilt or rejustified. In contrast, more solid resources can be re-recognized
quickly.

In Section 3.3, Heuristics in action, I explore these heuristics in an example from
damped harmonic motion. In Chapter 4, Coordinate Systems, I use them again in
discussing students’ coordinate system choices for simple pendula.

3.2.1 Frames and Framing

It is worth comparing plasticity to other theoretical constructs that are associa-
ted with Resource Theory. Hammer and collaborators[89, 86] have described the
process of recognizing which resources to use within a setting as “framing,” building
on Minsky[68] and Tannen[117], who use similar but distinct definitions for frames.
As a theory, Frames deal with “structures of expectation”[117]: the way participants
in a situation figure out “what’s going on here?” Framing is ubiquitous: every situ-
ation must be framed in order to make any sense of it. The canonical example from
framing literature involves monkeys biting each other. Framed as play, a bite may
elicit other playful behavior. Framed as aggression, it may start a fight[117].

3.2.1.1 A note on terminology

To confuse matters, framing an event can mean either building a frame of in-
terpretation or applying a knowledge schema in building that interpretation. To
help clarify, I use “framing” (gerund form) to mean an activity where one or more
knowledge schemata are recruited to build an interpretation of a situation; “to
frame” (verb form) is to recruit said schemata.
Tannen divides framing theories into two groups, the expectation of what may happen in general, and the interpretation of what is happening now. She terms the former group the “knowledge schema” and the latter the “frame of interpretation.”

In the literature, there is an alluring and confusing tendency to refer to “frames” as nouns which could mean either the frame of interpretation (which recruits schemata) or the currently active knowledge schema(s) themselves. I use “frame” (noun form) to mean Tannen’s frame of interpretation.

Where possible, I prefer to use the gerund or verb forms, to encourage the idea of framing as an activity, in accordance with Hammer[89].

3.2.1.2 Knowledge schema and framing

A knowledge schema is a locally coherent group of resources which may be applied to specific problems. In terms of RBC, the constituents of a knowledge schema have been sufficiently reified that they apply across problems and do not refer to a specific problem. They can be recognized. In the problem of a ball rolling downhill, conservation, algebra, and energy resources may activate to solve for the velocity at the bottom. Conservation, algebra, and energy come from knowledge schemata. These knowledge schemata, which have been called frames, do not have an inherent grain size: they may be very large (like a lecture course), very small (like a sub-problem), or somewhere in between.

A frame of interpretation is the active bringing together of one or several knowledge schemata in an effort to understand and work with a situation. In that sense, framing can be thought of as a build-with action, instantiating resources to deal with the specifics of one problem. In the ball rolling down the hill example, the application of the resources to the problem is a result of framing.

Framing, as an activity, is restricted to the here-and-now of current problems and contexts. Just as multiple problems may be present in a given situation, multiple
framing activities may also be present in the same person. Tannen[117] uses an example of a doctor performing an examination on a child, for later presentation to more junior doctors (residents). The doctor uses an examination frame, complete with specific linguistic register and vocabulary, to report findings to his colleagues. However, he also uses a playful frame to interact with the child, looking for “a monkey in [her] ear.” This frame has a different vocabulary and register, and the doctor switches between the two frames repeatedly. Each frame is in answer to a different problem: the examination frame reports findings to colleagues, while the play frame encourages child to submit to examination.4

For the student interactions described in this thesis, two frames are active for the TA: a frame for helping students build physics understanding, and a frame for moderating a smoothly functioning group. In the former frame, the TA addresses physics content-related resources. In the latter, she uses social resources to encourage participation by all members, rein in gossip, control the pacing, etc.

3.2.2 Framing and Plasticity

Framing and plasticity are not the same. Framing is an activity which recruits resources to address problems or situations at hand. Plasticity tracks resources over time, looking at how inter- and intra- resource links develop in strength and availability. Plasticity is a property of resources; framing is an activity using resources.

Elby[118] claims framing is meant to show the development of resources, though this development is not detailed in his literature on the subject[89]. Because framing is meant to bring together locally coherent sets of resources, a frame might function like a proto-resource. Over time, the proto-resource may compile to produce an actual resource, ready to be activated as-is. A medical student learning how

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4Tannen argues that a third frame, for dealing with the child’s mother, who is present at the examination, is also active. The activation of this third frame, while relevant to the interaction as a whole, brings extraneous detail here.
to examine patients might have to deliberately medicalize her words because using *medical terminology* is not readily recruited into her nascent examination frame. Over time, using *medical terminology* becomes commonplace; the frame readily recruits it. Framing a situation as “an exam” requires use of *medical terminology*. In this sense, the plasticity continuum might be applicable to frames as well as resources.

Of course, beginning medical students are less adept than doctors at using medical terminology in general, not just in the context of physical exams; one purpose of medical school is to learn terms used in the practice of medicine. Both their exam frame and their medical terminology resources must develop as they become doctors. However, just because the exam frame recruits *medical terminology* does not mean that it is the only frame which does so: doctors also use *medical terminology* when discussing treatments with their colleagues. Therefore, the *medical terminology* resource can develop independent of the exam frame, and it can also develop alongside it.

### 3.3 Heuristics in action: an example

To illustrate the plasticity heuristics and to provide a touchstone example, I present a student discussion from the context of critically damped harmonic motion in an intermediate mechanics class. The students are working with a TA on a homework problem to derive an expression for critically damped harmonic motion.\(^5\) The first student, “Gina,” is soft-spoken but completely correct in her explanations of the mathematics and physics involved. She has already solved the problem at hand, writing her solution on a whiteboard with little elaboration. The second student, “Bill,” is less agile in his descriptions and more aggressive in his manner. I excerpt one episode from the derivation to illustrate plasticity and some of its

\(^5\)For more details of the research setting, refer to Appendix A, Methods.
Figure 3.1  A mass on a spring undergoing damped harmonic motion. At the instant shown, the mass is moving to the left and the damping force is to the right. The mass is to the left of its equilibrium position; the spring is compressed and pushing outwards to the right.

limitations. The episode, named “Forcesign,” concerns Bill’s successful reasoning for determining the signs of forces in damped harmonic motion.⁶

3.3.1 Forcesign

Bill works through the sign of a velocity-dependent drag force in damped harmonic motion (Figure 3.1). Immediately prior, Bill asks to go over why the drag and spring forces are negative in the vector statement of Newton’s Second Law:

\[ \Sigma \mathbf{F} = m \mathbf{a} \]  \hspace{1cm} (3.1)
\[ -k \mathbf{x} - c \mathbf{v} = m \mathbf{a} \]  \hspace{1cm} (3.2)

On his first attempt, he is unable to explain why the forces are negative, and he confuses himself while trying to figure it out. He starts over, explaining that the drag force opposes velocity and the spring force opposes the displacement, but he is unable to connect his description to the algebraic signs of the forces. While correct, his explanation confuses him again.

To refocus discussion, the TA asks about the \(-cv\) force first, holding the\(-kx\) force for later. On his third attempt (reproduced below), Bill’s explanation is longer, and

⁶A complete transcript is available in Appendix B, Transcripts, Section B.1.
he considers leftwards movement and rightwards movement separately. He references an implied coordinate system in which the positive direction is to the right. His explanation takes 37 seconds.\footnote{The line numbering in all clips is consistent with the section of Appendix B from which they are excepted. In this clip, the line numbering is consistent with Appendix B.1.}

TA 0:02:23 So if we think about the cv term, why is it minus cv?

Bill 0:02:27 cv... (writes -cv) cv... um... before...(draws equilibrium line and base) this velocity is going to be this way (draws arrow pointing right) and the air resistance is going to be that way (draws arrow pointing left). Alright, so the velocity is positive and the force is negative and when the velocity becomes negative (draws arrow pointing left) the force is positive (draws arrow pointing right) so it changes the sign around. Of this (points at -cv). In here. Alright?

Is that right? If this is the...

Bill’s third attempt is correct, complete, and does not confuse him. After figuring out the sign of the air resistance force, discussion moves to reversing the implied coordinate system and figuring out the sign of the spring force. When finding the sign of the spring force, Bill employs a similar argument, but he uses it much more readily and facilely. For a minute analysis of the argument and its implications about the plasticity of two resources, forcesign and coordinate systems, I first examine Bill’s successful reasoning about the air resistance force.

3.3.2 Plasticity Analysis

In this chain of reasoning, a request for reasoning is followed by elaborative sense-making and checks for consistency. It finishes with an optional appeal for group consensus.
Bill uses this chain five times in a productive six-minute episode, explicitly asking for agreement four times, and explicitly receiving it three times. See lines 9, 61, 74, 106 (discussed in detail in Section 3.3.3), and 115 in Appendix B.1 for Bill’s chains. Other students in other clips also use the chain to formulate and explain their reasoning. For examples, see line 262 in B.2, lines 36, 181, and 280 in B.3, and lines 165, 197, 260 in B.4.

In this example of the chain, we learn about the plasticity of two resources, *forcesign* and *coordinate systems*. In the next example, Section 3.3.3, we learn about how plasticity changes in Bill’s extended use of *forcesign*.

### 3.3.2.1 Request for reasoning

The chain starts when one participant (here, the TA) asks another (here, Bill) about a previous statement, focusing the discussion (line 59). The chain does not need to start with a refocusing question; it can also start with a request for a definition or a statement that an existing definition is incomplete.

### 3.3.2.2 Sense-making: elaboration

Bill responds by elaborating on his previous statement, making sense of the physics as he goes (lines 60–66). His hesitancy at the start (line 60) (which is more apparent in the video than the transcript), together with his earlier difficulties with these questions, signify that he is making sense of the situation as he goes, rather than repeating a pat explanation. I interpret Bill’s sense-making as performing a build-with action. His sense-making indicates that his ideas about the signs of these forces are not solid – they are neither predetermined nor readily available. Furthermore, the detail of his description indicates that *forcesign* is plastic (plasticity heuristic P3, elaboration) to him.
3.3.2.3 Sense-making: consistency check

Bill nests a consistency check within his elaboration (lines 63-66). By so doing, he explicitly tests if these newer ideas about the sign of the air resistance force are consistent with differently articulated previous work (plasticity heuristic P2, recency) involving coordinate systems and directionality. Because Bill is facile enough with these ideas to test their consistency, they are sufficiently compiled to be resources, not mere fluid ideas. However, because he needs to test explicitly, forcesign is plastic to him (plasticity heuristic P3, elaboration).

3.3.2.4 Justification through activity

Bill further justifies his response through reference to an activity: choosing and using a coordinate system (lines 63-65) (plasticity heuristic P1, ease of use). His tacit use of a coordinate system as justification implies that Bill’s coordinate systems resource is more solid than his forcesign one (plasticity heuristic P2, recency). The implied nature of his coordinate system is further evidence of its solidity (plasticity heuristic P3, elaboration). Because he does not explore the structure of coordinate systems here, merely referring to them, we cannot conclude very much about coordinate systems’ structure. A further discussion of resource unpacking is available in Chapter 4, Coordinate Systems.

3.3.2.5 Social norm: agreement

Bill finishes his chain with an explicit social call for agreement (line 67), signaling that he sees his reasoning as sufficient. Because his call is explicit, he does not see his reasoning as inherently self-obvious, further implying that his forcesign resource is plastic.

The chain does not always conclude with an explicit verbal call for agreement. Sometimes, the other participants voice their assent without the call. At other
times, the chain simply ends without verbal agreement. From the video data we have, which show a top-down view of the work surface and the participants hands, but not their faces, we cannot tell if non-verbal consensus is reached. Had we frontal views, we could capture more gestures and perhaps evidence of non-verbal consensus.\(^8\)

### 3.3.3 Extended use

The scope of this interaction – 37 seconds within a six-minute episode – is also quite small. In that short time frame, we see evidence of some resources’ plasticity, but we cannot expect to see their level of plasticity change. Furthermore, we cannot expect to see plasticity heuristic P5 (extended use) in such a short time frame. In the longer episode, Bill reuses the chain to reason about the sign of the spring force, \(-kx\), and to reverse his tacit coordinate system, making leftwards the positive direction. Each time, his reasoning is shorter and less verbally detailed, indicating solidification – an aspect of constructing.

After figuring out the air resistance force, the TA proposes reversing the coordinate system, so that positive is to the left. Bill and Gina both correctly reason that the sign of the force would still be negative. The TA then asks about the sign of the spring force, \(-kx\). In his reply, Bill quickly reasons through the sign of the spring force.

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\(^8\)The top-down view is more productive for this study because it captures what students write better. For a full discussion of research methods, including data collections, refer to Appendix A, Methods.
Bill 0:04:41 Um... When this is positive (indicates right side of diagram) its always... its always positive on this side (right) and the force is always negative, as it should be. So it should always be negative since k is positive and x is positive. And then when its on this side (left) the force is always going to be in the positive direction and then... this is the (unclear). Force... Okay.

Bill reasons correctly, but trails off at the end, leaving the TA confused. When the TA asks Bill to elaborate, he responds, “No. I get it.”, indicating that his sense-making is complete and there is no need to continue. Desirous of clearer data, the TA asks Bill to explain it to her, which he does. In this analysis, I focus on Bill’s first explanation.

Just as in his first episode, Bill uses the same chain of reasoning. However, this time he is building with recently activated resources, rather than activating as he goes. Furthermore, his reasoning describes his prior conclusions: “as it should be” indicates that he already believes the spring force to be always negative. In this second example, he uses his coordinate system more thoroughly to explain the forces, indicating that forcesign and coordinate systems are now more tightly bound and that forcesign is more solid. Finally, Bill’s explicit rejection of a social call for agreement at the end of his reasoning is further evidence that his forcesign resource is more solid here. For a chart of the plasticity of these resources for Bill, see Figure 3.2.

This solidification of resources with use is consistent with results from memory research, which holds that memories more frequently and recently accessed are easier to access in the future. Furthermore, a connectionist view of memory, for example, holds that memories which are well-connected to other memories are more easily
Air resistance force

Spring force

Figure 3.2 A comparison of the plasticity for *forcesign* and *coordinate systems* for Bill.

On top are his *forcesign* and *coordinate systems* for the air resistance reasoning. After reasoning about the spring force, his *forcesign* solidifies.

recalled than ones that are not so well-connected. By having multiple retrieval cues, the memory is more likely to be retrieved. Thus, to better learn new information, it is advantageous to blend it into an existing structure, rather than blindly memorize lists of facts. A Connectionist views a memory as not just a collection of nodes: the links between those nodes are at least as important as the nodes themselves. This emphasis on links is in agreement with resources, and is illustrated here in Bill’s tighter binding of *forcesign* and *coordinate systems*.

### 3.3.4 Discussion

The scope of this mechanics problem – the sign of the forces in damped harmonic motion – is extremely small. However, it is often difficult for sophomore level physics majors and therefore strongly emphasized in the curriculum and homework. This interaction starts because Bill recognizes both his own confusion and the problem’s importance to the course. Perhaps in other settings, different contexts would lead
to different resources’ activation, and this chain may not be employed. For example, had Bill activated his knowledge-from-authority epistemological resources instead of his knowledge-as-invented-stuff resources[17], he might have responded to the TA’s request for reasoning very differently. He might have responded that the damping term is negative because the professor said so. Such a response precludes our observation of the extensive sense-making used in the chain. More constraints of the use of plasticity are discussed in Section 3.5, Some limitations of plasticity.

3.4 Toulmin’s argumentation structure

Bill uses a reasonable argument for deciding on the sign of each force in the differential equation. In other fields, the structure of reasonable arguments has been studied extensively. Using the force sign example as a touchstone, it is useful to compare Toulmin’s seminal argumentation structure to the plasticity heuristics.

Toulmin[119] proposed that reasonable, natural arguments (“practical arguments”) have three necessary elements and three optional ones:

Claim: conclusions whose merit must be established.

Data: the facts to which we appeal as a foundation for the claim.

Warrant: the statement authorizing the connection between the data and the claim.

Backing: credentials designed to certify the statement in the warrant “Experts agree…”

Rebuttal: statements which illustrate any restrictions on the applications of the claim. “Side effects may include…”

Qualifier: words or phrases meant to express the speaker’s degree of force or certainty concerning the claim. Waffles or hedge words.

The most essential pieces to any argument are the claim, data, and warrant; the others may be employed as the speaker and situation merit. The warrant may
be implicit or explicit; the data and the claim are most often explicit. According to Toulmin, the strength of any argument hinges on the strength of the weakest warrant; however, Toulmin was concerned primarily with moral and ethical arguments and the revivification of casuistry (a middle ground between absolutism and relativism).[120] As extended into communication[121], his structure has been applied to a wide variety of arguments in non-science[122, 123] and scientific[124] regimes. Recently, Toulmin’s structure has been applied to mathematical proofs and arguments[125] and to student arguments in mathematics classes[126].

Toulmin’s structure can be applied to the chain of reasoning that Bill employs to figure out the sign of the forces. Using Toulmin’s structure, Bill’s claim is that the air resistance force must be negative. His data are the two cases of positive and negative velocity and force, and the implied warrant is that coordinate systems are an appropriate tool to analyze the situation.

I connect this chain to Toulmin’s structure more generally. Claims are plastic by P3 (elaboration) and P4 (justification) because they require the use of the rest of the argument to support them. Similarly, data are solid by P1 (ease of use) and P4 (justification) because they form the supporting evidence.

An implied warrant is evidence for solidity by P3 (elaboration). Extensive negative qualifier use (“possibly,” “might be,” “could”) may signify plasticity by P4 (justification), but it may also signify that the data and the claim are not well connected by the warrant. Toulmin’s structure does not address P5 (extended use).

Toulmin’s work and its extensions in communication science show that this argument structure is common. However, they focus on the degree to which an argument supports a conclusion, not the means by which speakers form meaning.

Rasmussen and Stephan[127] extended Toulmin’s work into mathematics classrooms to look at communities of learners making and sharing meaning. They pro-
pose two criteria to identify ideas for which the learners function as if the idea were shared. An *as if shared* idea is one where

1. the backings and/or warrants for an argumentation no longer appear in students’ explanations; the mathematical idea expressed in the core of the argument stands as self-evident.

2. any of the four parts of an argument shifts position within subsequent arguments and is unchallenged.

These two criteria bear a marked resemblance to plasticity heuristics P3 (elaboration) and P4 (justification). Rasmussen’s method can also be used in physics classrooms or other places where people engage in public argument.

An interesting property of Rasmussen’s method is that it holds implications for how evidence of changes in plasticity might be found. As ideas – resources – become treated *as if shared*, they become more solid. This solidification may not last beyond the group shared environment, or may only persist partially. A discussion of resource solidification, with extended example, is available in Chapter 4, Coordinate Systems.

### 3.5 Some limitations of plasticity

Plasticity is not for everything. There are both theoretical and experimental limits on plasticity.

To zeroth order, plasticity is a theory for how ideas develop. Thus, its applicability is limited in situations where ideas should not – and do not – develop. For example, plasticity-based analysis is unhelpful in routine exercises: all ideas present start out solid, and remain solid through the course of the problem. On the other end of the spectrum, in wholly novel situations, students may be too stymied to
make much sense of the situation. Their resource graphs may be too sparse, or their
sense-making too fractured, to make plasticity-based analysis fruitful.

As a theory for how ideas develop, plasticity is only tangentially related to the-
ories of how to teach. Plasticity does not hold specific implications for instruction,
other than to generally say that the role of teachers is to encourage the solidifica-
tion of correct ideas and discourage the solidification of incorrect ones. Educators
are often concerned with correct or incorrect ideas. The plasticity of a resource is
independent of its veracity, and plasticity is not a tool for evaluating correctness.

Furthermore, one of the base assumptions of Resource Theory – and thus of
plasticity – is that individuals (not groups) hold ideas. Thus, sociocognitive models
of learning may be too different from plasticity to benefit much from it. However,
frames are socially negotiated, and plasticity may be used to describe their devel-
opment.

A marker of a resource becoming more solid is that the user doesn’t to perform
consistency checks with typical use. These consistency checks can be internally or
externally motivated. Sometimes they’re performed internally and silently, in which
case they are difficult to observe. A question about consistency may serve s to make
them verbal, or it may force them to exist in the first place. Experimentally, it can
be difficult to tell if the consistency check was externally motivated or internally
motivated (but silent) and then voiced in response to a question. Furthermore,
it’s not clear from either a theoretical or experimental standpoint how an expert-
coherent check would necessarily be different from a novice-deriving check (except
perhaps in terms of speed, but even that’s not a perfect measure), especially since
expert can rederive things as a means of checking coherence. These problems with
measuring consistency checks illustrate why the plasticity heuristics are heuristics,
and not steadfast rules.
Plasticity does not extend Resources farther into neuro-cognitive models. The name, “plasticity”, harkens to brain plasticity, but I do not mean to imply that more plastic ideas are more easily learned or that solid resources have myelinized. Even further afield are models which have nothing to do with learning, such as alternative energy models for electrical generation.\footnote{Even though those models may share some of the same terms (resource, conservation, energy, plastic...), they mean them quite differently.}

Because plasticity analysis relies heavily on how people reason, it is difficult or inappropriate to use when their reasoning is hidden. For example, it can be difficult to use plasticity in analyzing written work because students do not often write down thought processes, just solutions. Similarly, plasticity-based analysis requires that people actively figure out a situation, rather than defer thinking to authority. Therefore, it may be difficult to use plasticity-based analyses on episodes where the TA states conventions or where a lecturer presents a solution. These episodes have educational merit of their own, but plasticity is not the tool to gauge that merit. Because student reasoning in these kinds of situations is hidden or absent, these constraints are experimental in nature.

3.6 Summary

In this chapter, I introduced a continuum, Plasticity, for detailing the development of resources, which blends elements of Process/Object and Cognitive Science with Resource Theory. The name evokes brain plasticity and myelination – markers of learning power and reasoning speed, respectively – and materials plasticity and solidity – with their attendant properties, deformability and stability. In the plasticity continuum, the two directions are more plastic and more solid. More solid resources are more durable and more connected to other resources. Users tend to be more committed to them because reasoning with them has been fruitful in the
past. Similarly, users tend not to perform consistency checks on them any more. In contrast, more plastic resources need to be tested against the existing network more often, as user forge links between them and other resources.

To help ground that theoretical work, I explored an extended example from damped harmonic motion. I found that Bill’s coordinate systems resource was much more solid than his forcesign resource, but that over the course of several minutes, forcesign became more solid. Following, I compared the plasticity analysis of Bill’s chain of reasoning to Toulmin’s argument structure, noting that the two analyses bear some markers in common. By showing how the plasticity of resources changes with time, Resource Theory can now account for learning processes instead of simply states of knowledge.

However, some open questions with plasticity remain. In the next chapter, I address issues of resource grain size and durability in a more substantial manner than were possible in the short forcesign example.
Chapter 4

COORDINATE SYSTEMS

In Chapter 2, Theoretical Frameworks, I chose a theoretical framework from five traditions, elaborated on the properties of resources, and presented four heuristics for finding them. One property of resources is that they are nestable: one resource may contain graphs of other resources. As researchers, therefore, we choose a level of specificity to examine, noting that other levels are possible and may yield interesting results.

In Chapter 3, Plasticity, I introduced a continuum, Plasticity, for detailing the development of resources. To help ground that theoretical work, I explored an extended example from damped harmonic motion. In careful and detailed analysis, I found that Bill’s coordinate systems resource was much more solid than his forcesign resource, but that over the course of several minutes, forcesign became more solid. Through the plasticity continuum, Resource Theory can now account for learning processes (as opposed to discrete states).

Coordinate Systems is not, of course, primitive to Bill: it nests within it other resources. However, in the forcesign example, he does not seem to explore coordinate systems’ inner structure and instead simply uses it. For this reason, coordinate systems can be thought of as a resource. At the same time, the resource is not primitive because it can be explored in some situations. In many other situations in Intermediate Mechanics and other classes, students explicitly consider coordinate systems, their choice, and their use (see Section 4.2, Coordinate Systems in Intermediate Mechanics, for more details). Thus, coordinate systems is an excellent resource to track over time. In this chapter, I describe the structure of coordinate systems and give examples of students’ structure and activations changing within coordinate
systems. In so doing, I address issues of resource grain size and durability in a more substantial manner than were possible in the short forceesign example.

4.1 Unpacking coordinate systems

In considering all of the details which compose various coordinate systems, and their use and properties, it becomes expedient to break the coordinate systems resource into three subgraphs of resources nested within it. This division is largely to aid researchers’ organization, as any task tends to pull resources from each of the three subgraphs. In choosing the three categories, I have classified resources by the role they fulfill as users activate them, as opposed to the kinds of situations in which they activate. Other schemes are possible, and I use one of them in Section 4.3.3.

The three subgraphs are: properties resources, which describe general properties that coordinate systems bear; use resources, which describe when to use coordinate systems and which coordinate systems to use; and case resources, which hold the specifics of given coordinate systems. An example of resources in each subgraph is available in Table 4.1, with further detail on the case graph in Tables 4.2 and 4.3. Some quick examples of combinations are available in Table 4.4. The resources in these tables are resources because users can activate them (resource heuristic R4, activatable) without reference to any internal structure (resource heuristic R2, referable), and they can do so repeatably (resource heuristic R1, reusable). In naming them, I have used resource heuristic R3, nameable. In keeping with the nestable nature of resources, I have tried to keep these resources at a similar grain size. Table 4.4 shows some small combinations of these resources to make other resources related to coordinate systems.

The exact breakdown of which resources belong in which sub-graphs, as well as the inter- and intra-subgraph connection details, are user specific and time specific.
Table 4.1 Subgraphs in *Coordinate Systems*

<table>
<thead>
<tr>
<th>Properties</th>
<th>directionality</th>
<th>positive and negative, forward and backward</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>orthogonality</td>
<td>each coordinate cannot be obtained through linear combination of other coordinates in the same system.</td>
</tr>
<tr>
<td>span</td>
<td></td>
<td>a set of all coordinates expresses all possible dimensions of the space.</td>
</tr>
<tr>
<td>equivalency</td>
<td></td>
<td>different coordinate systems are interchangeable quantities can be measured and labeled.</td>
</tr>
<tr>
<td>value</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Use</th>
<th>choice</th>
<th>coordinate systems must be chosen</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>explicitness</td>
<td>use may be implicit or explicit</td>
</tr>
<tr>
<td></td>
<td>natural</td>
<td>“preferred” coordinate system based on geometry</td>
</tr>
<tr>
<td></td>
<td>ease</td>
<td>“preferred” coordinate system based on calculational ease</td>
</tr>
<tr>
<td></td>
<td>arbitrariness</td>
<td>choice of coordinate system is not predetermined</td>
</tr>
<tr>
<td></td>
<td>consistency</td>
<td>Within a problem, coordinate systems should not change</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>Locational</th>
<th>Systems where all of the coordinates refer to positions. Usually the answer to “what coordinate system are you using?”</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Usually used for diagrams and pictures. Systems where some or all coordinates are not positions. Frequently (though not exclusively) used for graphs.</td>
</tr>
</tbody>
</table>

However, naming the one possible set of components and their interplay gives a baseline against which users’ ideas can be tested.

Using this breakdown of subgraphs within *coordinate systems*, it is possible to examine which resources activate in given situations and show *intra-coordinate systems* linkages. It is unreasonable to expect that all of these resources would activate in every episode; typically, only a few need be active, depending on the context.

To explore these resources, consider a ball rolling down a hill. One possible coordinate system is aligned parallel to the hill, positive downwards and zero at the top of the hill. In choosing this coordinate system, you activated *directionality* to set the positive direction and *value* to set the zero at the top of the hill. You also
Table 4.2  Some resources in the locational subgraph

<table>
<thead>
<tr>
<th>Resource</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cartesian</td>
<td>A rectilinear coordinate system (x, y, z)</td>
</tr>
<tr>
<td>polar</td>
<td>A circular coordinate system in two dimensions (r, θ)</td>
</tr>
<tr>
<td>spherical</td>
<td>A circular coordinate system in three dimensions (r, θ, φ)</td>
</tr>
<tr>
<td>oblate spheroidal</td>
<td>A three dimensional coordinate system based on a rotation of an ellipse (µ, ν, φ)</td>
</tr>
<tr>
<td>numberline</td>
<td>A one-dimensional coordinate system</td>
</tr>
<tr>
<td>diagrammatic</td>
<td>Used to visually answer the question “Where is...?”</td>
</tr>
</tbody>
</table>

Table 4.3  Some resources in the non-locational subgraph

<table>
<thead>
<tr>
<th>Resource</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>spaces</td>
<td>phase space, vector space, state space, etc.</td>
</tr>
<tr>
<td>Note</td>
<td>while these coordinates are not necessarily positions, the language used to describe their systems is location-like.</td>
</tr>
<tr>
<td>graphs</td>
<td>Often Cartesian representation of one or more non-positional variables</td>
</tr>
</tbody>
</table>

– likely implicitly – chose to use one coordinate à la numberline because only one coordinate is needed to span the ball’s likely motion.

Of course, this coordinate system is explicit, but it is not inherent in the motion of the ball: I chose it. Another possible coordinate system is to have the positive direction be vertically upwards, with zero at the bottom of the hill. The choice to use one or the other of these systems is somewhat arbitrary considering that the same information can be gleaned from either; they are equivalent. However, depending on the question at hand, it may be more calculationally easy to prefer the vertical coordinate (if I were using energy conservation, for example), or the parallel one (if I were finding the time to reach bottom, for example).

Table 4.4  Some pairs of resources within the coordinate systems.

<table>
<thead>
<tr>
<th>Pair</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>consistency and value</td>
<td>Origin: the place where all coordinates are zero.</td>
</tr>
<tr>
<td>span and orthogonality</td>
<td>Generalized coordinates</td>
</tr>
<tr>
<td>equivalency and ease</td>
<td>Coordinate transformations are sometimes desirable.</td>
</tr>
<tr>
<td>social agreement and choice</td>
<td>Convention: the system everyone usually uses.</td>
</tr>
</tbody>
</table>
4.2 Coordinate Systems in Intermediate Mechanics

Students do not discuss forcesign often in our Intermediate Mechanics course. However, one thread that runs through the entire course is the selection and application of coordinate systems.

In the first few weeks in a typical intermediate mechanics class, students study air resistance, often using the context of a ball thrown off a building. A common coordinate system to choose is Cartesian, with the two coordinate axes as horizontal and vertical. While the positive horizontal direction has a clear natural choice (the direction of horizontal motion), the vertical direction does not have an obvious clear natural or easy choice. Students alternately consider up and down to be positive.

Later, they study harmonic motion (damped and undamped), often using a mass on a spring. A common one-dimensional coordinate system uses the equilibrium position of the mass as the zero of the coordinate. A common problem in introductory classes is to verify that that this choice is consistent with their formulation of Hooke’s Law, $F = -kx$, by effecting a coordinate transformation. In both the introductory and intermediate mechanics classes, students consider the effects of switching the positive direction. In Chapter 3, Plasticity, I discuss how one student, Bill, reasons through this reversal.

Towards the end of the course, students may use Lagrangians and generalized coordinates to solve problems ranging from modified Atwood’s machines to coupled oscillators; a unit on accelerating reference frames holds explicit work on translating between two frames (and thus, two coordinate systems). Their discussions of angular motion and orbits include discussion of polar coordinates.

Upon starting the course, students have a strong preference to use a Cartesian coordinate system where the positive directions are up and to the right, because this coordinate system is the most solid to them and other coordinate systems are often
quite plastic. However, many problems in physics can be made simpler through
the use of other coordinate systems. For example, finding the position of a simple
pendulum as function of time is simplest using polar coordinates. Developing and
choosing the appropriate coordinates for a problem becomes more important as
these students develop as physicists.

In small group interviews at two points during the semester, we asked three
groups of students to define a coordinate system and set up the equations of motion
for a simple pendulum. We also ask students to solve the problem in a HHS. Though,
chronologically, the HHS precedes the minviews, here the miniview data is presented
first. The miniview data do not involve any of the students from the HHS, so there
is no risk of seeing a particular student out of time order. The miniview students
consider coordinate system choice at a larger grain size than the HHS students do.
Ordering the sections this way continues the theme started in Chapter 3, Plasticity,
of going from large to small grain size.

4.3 Simple Pendulum

To investigate students’ developing understanding of coordinate systems, as well
as other questions, we collect video data from the Homework Help Sessions, from
weekly small group short interviews, and from class discussion. We also collect
written data in the form of ungraded (post-lecture, pre-tutorial) pretests, homework
assignments, and exams.

One problem stands out for in-depth study: the choice of coordinates for a simple
pendulum. A common touchstone problem in mechanics, the simple pendulum
is first encountered as an example of simple harmonic motion in a first semester
physics course. Thereafter, students encounter it again in intermediate mechanics,
in lab classes, in graduate mechanics, and (by analogy) in quantum mechanics.
Just like the Atwood’s machine, the simple pendulum enjoys many modifications. In introductory courses, these include changing its length and local gravity. In later courses they include turning it into a physical pendulum, as half of a coupled pendulum, as a device to measure local gravity, as an exercise in lifting simplifying assumptions, and as an analogy to quantum systems.

Before entering the Intermediate Mechanics class at the University of Maine, students have encountered the pendulum problem in some detail in their calculus-based introductory physics course. The pendulum is but one (relatively simple) example of harmonic motion, one of the most important models taught in an undergraduate physics major. However, it is only one model of the hundreds available in an introductory textbook. In a short unit on simple harmonic motion within an introductory mechanics course, the simple pendulum is usually only presented as an also-ran to the more simply modeled mass on a spring. There is a lot of mathematical sophistication inherent in the pendulum problem that the mass on a spring problem lacks. Two key operations which differ – the choice of coordinate system, and the simplifying assumption that $\sin \theta \approx \theta$ – are usually performed by the instructor (who, to his\(^1\) credit, is more interested in showing similarities than differences). Because of these confounding details, I expected students to be familiar with the problem but to have forgotten the specifics of the modeling they previously did.

Previous research\([110]\) on student understanding of simple pendula indicates that students at all levels have difficulty with determining the direction of the acceleration of the bob as it travels through the swing. My focus is not on a qualitative conceptual understanding, but on the mathematical interpretation of this problem, specifically a choice of coordinate system.

\(^1\)I use “his” because the University of Maine does not employ female instructors in introductory physics lecture courses.
4.3.1 A physicist’s solution

To solve for the position of the bob as a function of time, a physicist would first write Newton’s Second Law for the system, a vector second-order differential equation. The two forces on the bob are the tension force ($T$), radially upwards along the string, and the weight force ($W$), vertically downwards. Their sum is equal to the mass of the bob times the acceleration of the bob.

\[ \sum F = ma \]  \hspace{1cm} (4.1)
\[ W + T = ma \]  \hspace{1cm} (4.2)

The mass of the bob is constant; the acceleration changes continuously in both magnitude and direction.

The physicist would then choose a polar coordinate system as shown in Figure 4.1. This coordinate system takes advantage of the natural geometry and symmetry of the situation, and it is a calculationally easy choice which finesses qualitative problems in determining the magnitude and direction of the acceleration. With the coordinate system in place, the vector equation of motion can be split into two scalar differential equations, and then solved. As the focus of the interviews was on the coordinate system choice, the students were not expected to solve the differential equations.

![Figure 4.1 The forces on a simple pendulum, with a physicist’s polar coordinate system shown.](image-url)
Figure 4.2 A simple pendulum with two positions (left and right), two options for measuring $\theta$, and all reasonable choices $\hat{\theta}$.

To a physicist, $\hat{\theta}_1$ and $\hat{\theta}_3$ are identical, as are and $\hat{\theta}_2$ and $\hat{\theta}_4$. To a student, the unit vectors in each pair are not necessarily the same.

4.3.2 The students’ problems

In HHS, students are asked to solve for the position of the bob using both Newtonian and Langrangian methods. A necessary prerequisite to either method is choosing a coordinate system. So that students do not spend time figuring out the forces on the bob, and to predispose the students into thinking of force-based solutions, the forces on the bob are given diagrammatically.

In interviews, students work on (ostensibly) the same problem. In reality, I did not expect them to solve the differential equations, and students were directed to determine and discuss their coordinate system. So as not to predispose students into choosing a particular coordinate system, the forces are not described as being “vertically downwards” (weight force) or “radially inwards” (tension force). As can be expected from physicists’ sloppy distinctions between $\hat{\theta}$, $\theta$, and $\theta$ in ordinary speech, on occasion the students and the TAs are sloppy as well.

In the course of their discussions, students explore all possible directions for $\hat{\theta}$ and $\hat{r}$. Figures 4.3 and ?? show four options for each of $\hat{\theta}$ and $\hat{r}$, respectively, for both leftward and rightwards positions of the bob.
Figure 4.3 A simple pendulum with two positions (left and right), two options for measuring $\theta$, and all reasonable choices for $\hat{r}$. To a physicist, $\hat{r}_1$ and $\hat{r}_2$ are identical, as are $\hat{r}_3$ and $\hat{r}_4$. To a student, the unit vectors in each pair are not necessarily the same, though students seem to have greater agreement on $\hat{r}$ choices than on $\hat{\theta}$ choices.

4.3.3 Summary of common resource use

Because of the length and detail of the transcripts presented below, it is helpful to summarize which resources are used throughout the HHS and interviews. Some resources in Tables 4.1-4.4 are never observed. Others are quite common. In particular, in the case subgraph, only Cartesian and polar activate, as expected. These are often activated because they are natural to the geometry or easy to use mathematically. In addition, span arises in determining whether the chosen coordinate system can actually describe the entire system (and the space in which movement occurs) appropriately; directionality and value are frequently invoked to test cases. Other resources are also commonly activated in this problem, notably arbitrariness, equivalency, explicitness, and orthogonality. As an artifact of group problem solving, social agreement is also frequently activated in addition to epistemological resources, metacognitive, and problem solving resources. I do not dwell on non-coordinate-related resources in this chapter, other than to note their existence.

In choosing a coordinate system, different resources fulfill different goals. In general, social agree and arbitrary force a choice to be made; explicitness may make
this decision more apparent. Arbitrary and equivalency allow multiple options to be considered simultaneously, but consistency requires that one option reign over the length of a problem. Thus, the triad of arbitrary-equivalency-consistency requires that one coordinate system be chosen among several considered. This choice could be different for different people. If those people work together (or agree to compare results), however, social agreement requires that they use the same system.²

The arbitrary-equivalency-consistency triad allows options and forces a single choice among them, but it does not provide mechanism for which specific system should be chosen. Consistent, natural, orthogonality, span, and ease may mediate the details of which system is chosen. Not all of these may obviously activate in each situation or for every participant, and they may not activate at the same time.

In the case pictured in Figure 4.4, the decision is between polar and Cartesian. At a different grain size, the choice may be to decide the direction and value for a specific coordinate. In Miniviews, students focus on deciding between polar and Cartesian. In HHS, they work within polar to determine direction and value separately for each coordinate.

In this section, I have grouped resources by what task they perform as users choose a coordinate system. Roughly speaking, those categories form columns in Figure 4.4 or Figure 4.16. This classification scheme is different than the role-based one in Section 4.1. The role-based system is a gentler introduction to the resources; this system is more closely allied with data.

4.3.4 A physicist’s graph?

A complete treatment of which resources might help choose a coordinate system (and with which linkages) for an expert solving the pendulum seems like it might be

²Social agreement could also be with a problem statement (as described in the HHS) or with convention.
Figure 4.4 A general resource graph showing the coordinate system choosing process between polar and Cartesian, with some details omitted.

interesting. However, there are many reasons, both theoretical and experimental, why such a graph is neither practical nor desirable.

For an expert, there are a lot of resources and a lot of links. Visually, there would be so much information in the graph that unpacking the graph would be a lot of effort for not a lot of gain in understanding. Furthermore, if making a complete laundry list of resources is neither possible nor desirable, then making a complete graph of a complete list is doubly so. The point of the general graphs is not to have a canonical right answer against which all other answers are to be judged. Different connections are sufficient for different contexts (even within the same expert), and drawing all possible connections (an enormous task) obscures that optimization.

If a general graph for an expert is not possible, then another possible avenue is to interview an expert about which coordinate system to choose and why. Alternately, one could contrast an expert’s resource use in a quasi-natural situation (classroom
during lecture) to a more clinical one (interview). The different contexts would likely activate different resources, and two specific graphs could be built. Interviewing professional physicists is outside the scope of this study.

The point of the general graphs is to draw attention to some of the resources in play for some people so that later graphs can focus on how the connections between the two sides of the curly brace appear. They are more readable if I name the resources to the immediate left and right of the curly brace, but less readable if I include more resources.

4.4 Miniviews

Prompted by the rich data in HHS, we asked students to choose a coordinate system in small group interviews in Spring 2006. Three groups of students discussed this problem during Week 4 and Week 10 of the semester. One pair was clear and on task most of the time during both interviews.

The pair, “Derek” and “Wes,” volunteered to be interviewed together. Derek was a conscientious student who submitted thorough solutions to assigned problems. He started the semester averse to small-group tutorial work and finished a loyal supporter. In contrast, Wes rarely submitted complete solutions and had a poor work ethic. At times, he appears to enjoy being ornery. They were good friends and enjoyed mutually abusive banter. The playful tone of their interactions is not always evident in transcript.

In this section, I focus on only a few examples of resources which are closely tied to the pendulum problem. The case resources commonly activated in this problem are Cartesian and polar. These are often activated because they are natural to the geometry or easy to use mathematically. Finally, the issue of span arises in determining whether the chosen coordinate system can actually describe the entire
system (and the space in which movement occurs) appropriately. The decision of which to pick is arbitrary, though all participants should agree on a coordinate system. In the process of that agreement, it is common – though not necessary – that they make their coordinate system explicit. In this interaction, I focus on the interplay between Cartesian, polar, natural, ease, and span.

4.4.1 Defining a coordinate system, Week 4

In the first interview (Week 4), the TA presents the problem. Wes and Derek immediately launch into a discussion of Newton’s Second Law, its applicability, and energy considerations. After several minutes of discussion without reference to a coordinate system, the TA asks, “What coordinate system are you using?” (line 129) When her question is met with silence, she continues, “Let’s make one.”

139 Wes: (points) He loves this. We may not use it. (Draws cartesian coordinates) Jeezum crow, that’s an x.

141 6:34

Derek: Out of curiosity, why not use polar?

143 Wes: Eh?

Derek: Why not use polar? We’re dealing with angles that are changing?

145 Wes: We’re going to deal with T, we’re going to deal with this, L.

Derek: Right, that’s all constant, though.

147 Wes: Yes, and those are the only things in it. There’s not even mass in it.

149 Derek: Mass...technically...[inaudible].

Wes: Yeah.

3These line numbers are consistent with Appendix B, Transcripts, Section 2, Week 4 Miniview.
Derek: No it won’t.

7:00

TA: I, I’m, mmkay.

Derek: If we use a polar coordinate system, all the r...all the radii from
the various points, are just going to be constant. The only thing
that’s ever going to change in this is angle.

Wes: Radii? Yes. [inaudible].

Derek: Makes the math easier.

Wes: I’ll use the eraser, so my hands don’t get black. (erases). (draws
polar coordinates) Here’s theta, this is an r. Does it really matter?

Derek: No.

Wes: Good. Umm...We can define this though, right? Length?

TA: Yeah, you can call that L, that’s ok.

In line 139, Wes draws a Cartesian coordinate system where positive is up and
to the right. When he grumblingly redefines his system in response to Derek’s polar
suggestion,\textsuperscript{4} he relabels the y-axis as the r-axis, and defines the angle between the
x-axis and r-axis as $\theta$ (Figure 4.5). It is notable that some elements of Wes’s polar
system agree with mathematical convention: the two coordinates are $r$ and $\theta$, and
$\theta$ is measured counterclockwise from the left horizontal line.\textsuperscript{5}

\textsuperscript{4}The tension is not constant, in conflict with Derek’s claim in line 146. Note that I generally
do not comment on the correctness of the physics in their reasoning, unless it relates to resource
use and plasticity. The TA does not comment on the correctness in the Miniview because the
protocol cautioned her against getting stuck on the magnitudes of the forces.[110] See the protocol
in Appendix A, Section A.2.4.

\textsuperscript{5}A conventional physical choice for this problem measures $\theta$ counterclockwise from the down-
wards vertical line, a quarter-circle phase difference. Both choices are valid. That Wes chooses
mathematics convention, not physics convention, is not surprising given that his primary experi-
ence with polar coordinates has been in mathematics classes, not physics classes.
Figure 4.5 Two coordinate systems from Wes. At first, he draws the $x$ and $y$ coordinates. In response to Derek’s polar suggestion, he replaces $y$ with $r$, labels the angle between $r$ and $x$ as $\theta$, and erases $x$.

In this interaction, Wes chooses Cartesian because it is a natural choice. Derek argues that polar is more calculationally easy because only the angles change, not the radii. Because only one coordinate is changing (a statement of span), the system can be reduced to a numberline-like problem.

Is is notable that neither Wes nor Derek feel a need to explicitly define their zeros for either coordinate, and consider the coordinate system discussion closed at this point: Wes immediately moves on to ask the TA if they’re looking for the position as a function of time. Wes returns to his thoughts on finding position later in the miniview; I return to his thoughts in Section 4.4.2.

To return to discussing the coordinate system, the TA asks Wes and Derek to apply their coordinate system, which is drawn next to the sketch of the pendulum (as in Figure 4.5), to the sketch itself:

183 TA: Tell me more about this coordinate system you, you set up here. 

Wes: Just polar.

185 TA: Kay....What direction is the-
Wes: Positive.

TA: In this picture? Yeah, which way is it.

Wes: This is positive theta. Counterclockwise.

TA: Which way is this, r.

Wes: r is (gestures to the pendulum string on the diagram). If–what?

Derek: It really doesn’t make a difference how you define it.

The speed of Wes’s response, coupled with its brevity, indicates that he’s using polar whole, without extensively deriving directions and zeros for the two coordinates. Wes is working at the level of choosing among prebuilt coordinate systems rather than constructing a coordinate system from scratch. Wes labels the polar angle as measuring counterclockwise from horizontal to the position of the bob ($\theta_1$ in Figure 4.6). Just after this clip, both students continue, readily volunteering that $r$ should be measured outwards from the attachment point of the pendulum.\(^6\)

Derek’s statement at line 191 is an expression of arbitrariness: it doesn’t matter which direction is positive.

Because Wes’s definition of $\theta$ indicates a curved path, but unit vectors are always drawn as straight lines, the TA presses Wes to show the direction of $\theta$ at the instant shown, hoping that he will choose a direction tangent to the path. Wes demurs, asserting again that $\theta$ is “counterclockwise.”

I interpret this system-defining interchange as showing Derek and Wes using different coordinate systems, polar (because it is easy) for Derek and Cartesian (because it is natural) for Wes. Furthermore, polar coordinates are connected to physical examples for Derek. They are less connected for Wes, for whom Cartesian coordinates are “traditional” (line 259) for any given problem and polar coordinates

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\(^6\)The question of origin is an interesting one. I return to it in Section 4.4.3.
are not well defined. This indicates differing levels of plasticity: Cartesian coordinates are more solid for Wes, polar coordinates more plastic, and polar coordinates are more solid for Derek than they are for Wes. However, we do not yet have evidence to compare the plasticity of Derek’s polar resource to Wes’s Cartesian resource.

4.4.2 Position, time, and span, Week 4

After reciting positive directions for the TA, the students feel that their coordinate system definition is sufficient again. They quickly move on to solving the “real problem”: finding the position as a function of time. They discuss the the graph of position vs. time (“some sort of sinusoidal pattern” in line 230), whether mass is important, and whether they remember the formula for period.

The TA brings them back to discussing their coordinate system choices by asking them to draw their system on the board. Wes petulantly interjects with “Does it even matter?,” an expression of arbitrary. He seems to object to the TA’s continued harping on the matter of a coordinate system when he wants to move on to the “real problem.”

![Diagram](image)

Figure 4.6 Derek and Wes use four definitions of $\theta$ at different points in the two miniviews.
Contrast their choices with the variety in Figure 4.3 or the physicist choice presented in Figure 4.1.
Recall that after the coordinate system was first mentioned, and the pair decide on a polar system, Wes quickly moved to asking about the position as a function of time.

Wes: That’s good. So you would want an equation that is any position in terms of time.

TA: Well...the position of the...yeah.

Wes: The position of this–

TA: In terms of time, yeah.

Wes: Now, do you want position in terms of this way (gestures horizontally on the diagram)? Or this...

Derek: Do you want like x,y position?

TA: All, all I want to be able to do is tell you where it is–

Derek: It’s much easier to use angles.

TA: –at any time.

Wes: Yeah.

Derek: Hence, why it’s easier to use angles.

Wes: To just know where it is. Totally.

Wes’s words and gestures indicate that he is still thinking about Cartesian as the most natural system for this problem. In contrast, Derek strongly holds that “angles” – a reference to polar systems – are easier.

Wes returns to his question of position as a function of time later in the miniview, starting with his petulant comment about arbitrariness. Internally, Wes is still using Cartesian to describe this system.
Wes: I would say if we had it in the traditional x,y, I just don’t know where to go from...the start point.

Wes’s activation of *Cartesian* as the most *natural* system for any problem – a “traditional” or default system – is getting in the way of solving for the position as a function of time. The TA asks him to consider how to solve the problem using polar coordinates. Wes responds that he would “just break it down into friggin’, um, arc lengths. Kinematics equations with forces, and do some trig.” (line 264). Derek eggs him to do so, and Wes grabs the marker, narrating his ideas.\(^7\)

Wes: And work out all the math. Hey, I don’t want to work out all the math. [redacted] (grabs marker). Well, let’s say this is L and say this is here, and this is a triangle, and that’s a right angle, and that’s some other distance I’m not going to name, yet, cause I don’t feel like it. But...this is our theta up here. The sine theta...(writes). Yes, no?

Wes: Mmkay. And, uh, we say our starting height (pause) is whatever distance, L minus this. That’s our starting displacement from wherever it’s going to be at the bottom. Then I’d use energy and figure out what it’s speed would be at the bottom. But... we’ll stay away from energy, I guess. (pause) Cause my mind’s trying to be connected to...we want something to tell it’s position with respect to time. (pause. 15:00) [continues]

During Wes’s extended argument, Derek is largely silent and working privately on an unrelated problem. In analyzing Wes’s argument, I break the argument into

\(^7\)some intervening lines have been omitted; refer to Appendix B.2 for more detail.
three segments, the first two of which are shown in the clip above. In the first part (lines 267-280), Wes labels lengths, heights, and angles in the problem. In the omitted lines above, he seeks Derek’s social agreement on his naming scheme. In the third part of Wes’s argument (lines 280-285), he moves to trying to solve the problem using his preferred coordinate system, possibly using conservation of energy.

Wes’s diversion into energy is not just fleeting whim here. When the problem is presented to the pair again in Week 10, he brings up energy arguments again. Physicists find that energy conservation (as opposed to vector force summation) is an attractive solution method for many problems in physics. For a simple pendulum, energy is conserved as it swings to- and-fro. Because the string is not stretchy, the kinetic energy at the bottom of the swing is entirely converted to gravitational potential energy at the top. Solving for the speed of the bob as a function of position is therefore a trivial exercise. Using energy arguments, a natural coordinate system is Cartesian: gravitational potential energy, expressed as mgh, requires that one coordinate be vertical, and measured with positive up. Wes is most likely thinking of these arguments when he defines his “starting height” in line 278.

Energy conservation arguments are alluring, but energy arguments cannot be employed to solve for the position as a function of time. In the second part of Wes’s argument, he mentions that he would like to use energy, but then he gets stuck with that approach. Though he is not explicit, it may be that some of his difficulty stems from the inapplicability of energy arguments to this question of time. That is not the sole source of his difficulty with this problem, however, as he goes on to describe in the third part of his argument, included below.
Wes (15:00) [continued]: And if you want position in x,y, then, it’s gonna be...stupid to do. To try. It would be easier to do position as opposed to angle of displacement. (hands marker to Derek, who begins writing)

TA: Does uh–

Wes: Rest point.

TA: Does a position based on angle and how far you are away give you the same information as it’s x,y coordinates.

Wes: Well I can’t think of any way to put it in terms of– you want what’s this point. What are it’s coordinates at this point?

TA: ’Where is it?’ ’Where is it?’

Wes: But you’d have to do that in terms of coordinates, right?

TA: Sure.

Wes: Sure.

TA: That’s–

Wes: And, uh...so I would say I don’t know any functions that would give you out two parameters.

TA: Mnkay.

Wes: At the same time. [inaudible] I mean, you could base it on y and say ok and then plug it into a different equation and get your x.

TA: Okay.

Wes (16:00): If you want a specific point, I would almost want to take the route of what’s it’s angle of displacement. But,

TA: Would displacement tell you exactly where it was? At any time?
Wes: Huh? Well, no, you’d have to figure that out. But,

In this third part of Wes’s argument, Wes wonders if it would be smarter to measure position based on displacement from starting angle. The TA’s question in lines 290-291 is a pointed question of equivalence and span. Wes does not address the equivalence argument. Instead, he explains that he “[doesn’t] know any functions that would give you two parameters” and thus he can’t solve the two-dimensional problem using the “traditional x-y” coordinate system. He seems to have activated span as a relevant measure of modeling, but seems unable to connect ease and span together to point to his chosen coordinate system. The TA presses the point in line 307, asking if angular displacement is sufficient. Wes replies that he doesn’t know and that he’d “have to figure that out,” further evidence that Wes’s polar resource is not well connected to other resources. In particular, it seems connected to neither natural nor easy. Furthermore, it seems that while he has activated equivalence in order to consider using an alternate system, he is unable fully switch to that system because it is neither natural nor easy.

When the equivalence question is put to Derek in lines 327-329, he replies quickly that knowing $\theta$ as a function of is sufficient. The quickness of his reply and the matter-of-fact tone in which he delivers it are evidence that polar is more solid for Derek than it is for Wes. In contrast to Wes, Derek sees the span of polar and Cartesian coordinates as sufficient for this problem and sees polar coordinates as natural for this problem. I represent this description of Wes’s and Derek’s resource use in Figures 4.7 and 4.8. Note that one arrow is drawn to indicate the explicit lack of connection between Wes’s ease and Cartesian resources.
4.4.3 Origin

In all locational coordinate systems, locations are expressed in reference to the values of the coordinates. In a Cartesian system, the zeros of each coordinate (a statement of value) converge at one point (the origin), and that point can only be specified using all coordinates. Thus, origin is a combination of value and consistent, as shown in Table 4.4. In a polar system, at the point where the radial coordinate is zero (analogous to the origin in Cartesian coordinates), the angular coordinate may take any value. In that respect, this point is unique in polar coordinates, and Cartesian coordinates do not have an analogous point that can be wholly specified using only one coordinate. Asking where the origin is – and meaning the place where all coordinates have a value of zero – when using polar coordinates is not a fair question.
Figure 4.8 Wes’s resource graph for the week 4 miniview.
Note the explicit lack of connection between ease and Cartesian resources. for Wes, but not Derek. Only natural, ease, Cartesian, polar, and span are shown.

Wes and Derek both activate origin resources, as evidenced by their discussion starting at line 183 when the TA asks them to apply their coordinate system to the sketch of the pendulum. Immediately following the \( \theta \) direction and value discussion, they continue into a discussion of \( r \), specifying both the location for zero and direction.

Later, to further explain his span argument and equivalence confusion, Wes brings in a discussion of value. He restates his argument to Derek, who had been working privately the first time, reiterating that to get a “precise exact location in the x and y,” two equations are necessary, one for each coordinate. Each position is thus expressible in terms of the value of each coordinate. In polar, however, Wes has a problem with expressing locations:
Wes: But in polar, if you want it’s exact location, in space, you have to
have it in reference to something.

Derek: The origin.

Wes: So if you have it referenced to the origin, you know that this length
is going to be constant, which is r in this system, and so you could
say it’s anywhere there [in a circle of radius L].

Wes’s difficulty in expressing the “exact location” of the bob may be related to
the nature of the origin in polar coordinate systems. It seems here that he does
not view polar coordinates as being a proper coordinate system, further evidence of
polar’s plasticity to him.

It could be argued that Wes does not have a polar resource at all: the components
of a polar system are too weakly connected to call polar a resource for him. Such
an argument belies Wes’s original work in determining the coordinate system. In
line 184, he refers to his coordinate system as “just polar,” as if that is a sufficient
response to the TA’s request to tell her more about the system; earlier, he quickly
names and labels his coordinates (lines 159-160), r and θ. By resource heuristic R2
(referable), polar is a resource. However, by plasticity heuristics P1 (ease of use),
P3 (elaboration), and P5 (extended use) it is extremely plastic.

4.4.4 Revisiting polar coordinates, Week 10

In Week 10, the task in Figure 4.1 is posed to the students again. In the inter-
vening weeks, students have studied damped and driven harmonic motion in class,
and have been assigned a homework problem on the equation of motion for the pen-
dulum (derived using both Lagrangian and Newtonian methods). Their responses
typify their approaches to the class: Wes says that he “tried to use radians, but got
Figure 4.9 Two plasticity charts for the Week 4 miniview. Wes’s graph is on top, Derek’s below. Wes’s polar resource is more plastic than both his Cartesian one and Derek’s polar, but we don’t have enough evidence to learn how solid Derek’s polar is.

stuck and gave up”; Derek says that to solve this problem, he would just “assume a solution.”

When the TA asks them to define a coordinate system, Derek chooses a polar coordinate system for the same reasons he did in week 4. Wes once again chooses a Cartesian system. However, instead of choosing the standard system where positive is up and to the right as he did in week 4, he tailors his system to the problem at hand, defining positive down and to the right. The downward direction is consistent with the weight vector, showing a better match of coordinates to physical situation. Because he tailors his coordinate system to the problem, rather than automatically choosing a generic system, his Cartesian resource has become more solid.

At the TA’s prompting, Wes continues to write Newton’s Second Law for the system and starts to break the forces into components, defining $\theta$ as the angle between the horizontal and the position of the bob ($\theta_1$ in Figure 4.6). His choice of Cartesian coordinate system complicates the problem, and he gets stuck.
Derek uses Wes’s confusion as evidence that a Cartesian system is inappropriate. As illustration, he writes Newton’s Second Law and breaks it into r and θ components. Derek writes,

\[ \sum F_x = m \frac{d^2 \theta_x}{dt^2} \] (4.3)

\[ T \cos \theta = m \frac{d^2 \theta}{dt^2} \] (4.4)

After writing Equations 4.3 and 4.4, Derek reads them aloud. In reading them, he corrects himself: because θ is a coordinate in its own right, he does not need to use the subscript x. His equation, amended, reads:

\[ T \cos \theta = m \frac{d^2 \theta}{dt^2} \] (4.5)

This equation differs from the standard physics equation because the angle defined as θ is the complement of the typically chosen angle. Furthermore, it is dimensionally inconsistent: the left-hand side has units of force and the right-hand side has units of mass per time squared. These differences aside, Derek’s equation has the right form for the differential equation.

An equation in place, the TA again asks the students to label their coordinates on their diagram. Derek first copies over Wes’s definition of θ, (θ1 on Figure 4.6), then argues that by alternate interior angles, it is equal to θ2. The TA asks where θ is equal to zero, and Derek redefines θ to be θ3, the common and calculationally easy physicist response. When the TA asks the direction of θ at the instant shown, Derek argues that the bob is moving in a circular arc and that, at any point along the arc, θ is tangent to the arc. He draws \( \hat{\theta}_4 \). With all four θ definitions arrayed before him, Derek expresses doubt that he has a sensible answer.

I interpret the evidence from the week 10 group interview to indicate even more strongly that Wes’s polar resource is very plastic (Figure 4.10). It is not well connected to other resources, in particular with calculational ease. Derek, whose polar
Figure 4.10 A comparison of polar and Cartesian’s plasticity for Wes in weeks 4 and 10. Cartesian has become more solid as his Cartesian choices become tailored to the problem at hand, while his polar resource seems more plastic.

resource seemed more solid than Wes’s, shows evidence of problems with the ease of applying the natural coordinates for this problem, indicating that it is still plastic to him (Figure 4.11). I present resource graphs of Wes’s and Derek’s resource use in Figure 4.13.

4.4.5 Discussion

In this section, I applied the theoretical structure of resources and plasticity to help explain student reasoning about coordinate systems for a simple pendulum. The simple pendulum represents a canonical physics problem that nevertheless presents difficulties to students. I show resource graphs of two students applying resources that are parts of the coordinate systems resource, and used these graphs to indicate the level of plasticity of the different sub-resources within coordinate systems. Rather than illustrate all possible resources within coordinate systems, I focus on Derek and Wes’s use of natural, ease, Cartesian, polar, value and span. By
Figure 4.11  A comparison of the plasticity of Derek's polar resource in weeks 4 and 10.

In week 4, there was insufficient evidence to know how plastic polar is (only that it is more solid than Wes's). In Week 10, Derek’s derivation details indicate that it is plastic.

Figure 4.12  Derek’s resource graph for the week 10 miniview.
asking the same question in Weeks 4 and 10 in the semester, I uncover some changes in their resource use and the plasticity of these resources for these students in this context.

In particular, Wes persists in using an inappropriate Cartesian system despite professed knowledge of polar coordinates, indicating that Cartesian coordinates are quite solid to him. Furthermore, he and Derek must rederive (rather than recall) the details of the polar coordinate system, indicating that polar coordinates are plastic to them. Derek’s reasoning is more facile, indicating that his polar resource is more solid than Wes’s.

In the next section, the focus shifts from changing plasticity in different weeks of the semester to examining how students build a coordinate system in a similar problem, and how their construction indicates the plasticity of polar coordinates.
4.5 Dialing down the scale: HHS

In the HHS on 4 April 2005, students solve the pendulum problem in using both Newtonian and Lagrangian methods as a pair of homework problems. As homework problems, the task is more extensive than in miniviews. Students are to set up the problem, including choosing a coordinate system, set up equations of motion, and solve them. In this analysis, I focus on their coordinate system derivation and choices.

A HHS can span three hours, and the whole HHS is not relevant to a discussion of coordinate systems. For this thesis, two episodes are relevant. In the first, (coordinates-1), five students and a TA derive a polar coordinate system in preparation for using Newtonian methods. Following this episode, another student arrives and the participants split into two tables. In the second episode (r-forces), a subset of the participants write the sum of forces in each direction for the system, and it develops that different participants are using different definitions of $\hat{r}$.

In the miniviews, Wes and Derek focus on choosing between polar and Cartesian on the basis of which is more natural or easy. In contrast, students in the HHS readily choose polar, but then spend a long time deriving the direction and value for each coordinate. The HHS students operate on a different grain size than Wes and Derek, and their discussion illuminates more of the structure of coordinate systems on a finer scale as well as showing how intra-resource connections can be used to illustrate plasticity.

4.5.1 Participants

Initially, five students and a TA are present. The students are\(^8\):

\(^8\)Names are pseudonyms.
Ed Alone of his classmates, Ed was a secondary science education major. He brought a desire to understand the connections between physics and mathematics to the fore of many discussions, and was rarely satisfied with superficial manipulations. As a student, Ed’s work often reflected his own fractured understanding. Ed was a frequent participant in HHS.

Jessica Jessica was one of the stronger students in the class. Neither assertive like Rose nor effacing like Martine, her contributions to discussion were equal parts confusions and clarifications.

Rose Of all the students who participated in HHS, Rose has the strongest conceptual understanding of the mathematics involved. She was also one of the stronger students in the class, and quite opinionated about what directions to take in discussions.

Martine Martine was a quiet participant, with both a lilting voice and infrequent utterances, though a frequent presence at HHS. Her written work was weak.

Vivek Vivek seemed to have problems putting his ideas into English, and did not speak much at HHS, where he was an infrequent participant. His written class-work was weak, reflecting his language difficulties.

When a sixth student arrives, Ed, Jessica, and Vivek stay at the video table, while Rose and Martine join the newcomer at a second table with a second TA. Regrettably, there is no recording of the second table.

4.5.2 Coordinates-1

In this episode, participants work out the directions of \( \hat{r} \) and \( \hat{\theta} \). Jessica and Rose, the stronger students, lead the discussion. Martine and Ed provide a chorus of confusion which encourages Jessica and Rose to elaborate on their ideas; Vivek often acts as choragos.
4.5.2.1 Choosing a system

To focus discussion, the TA draws a pendulum and equilibrium line like the one pictured in Figure 4.1, but without the vectors shown. The TA asks the students, “How does my coordinate system go?” (lines 13-14), requiring that coordinate system use be explicit and noting that a coordinate system must be chosen. Vivek responds that “We can just choose what we want” (line 15), a clear expression of the arbitrary and choice resources. The TA responds that some choices are better than others, thereby encouraging students to use either (or both) of natural or ease. Vivek replies that an easy choice would be to “pick it so that ... you have the forces along the x’s or y” (line 18), indicating that he is implicitly using Cartesian as the natural choice. Jessica responds that “if we have thetas we should be using polar coordinates” (lines 20-21), an expression indicating that polar is consistent here – the students should agree with the problem statement. The group appears to switch to polar coordinates in response to her suggestion, and the apparent switch is made quickly (lines 22-23).

On first blush, it seems that this group makes the decision to use polar coordinates quickly and without much discussion. However, they make their decision based on different criteria than Wes and Derek do. Wes and Derek spend most of their miniviews (Section 4.4) discussing which system in the case subgraph is appropriate, using natural and ease to decide between polar and Cartesian. In contrast, the HHS students finesse that question with an appeal to authority: polar is appropriate because the answer (given by authority) must be consistent with the solution.

Later in the episode, the students have decided on a polar coordinate system when the bob is to the right of equilibrium. To test this agreement and understanding, the TA asks about the coordinate system if the bob were to be on the other
side, to the left of equilibrium (line 146). The students quickly agree that \( \hat{r} \) will still be radially outwards (line 148), but confusion ensues in the discussion of \( \hat{\theta} \) (starting at line 152).

Before the discussion starts in earnest, Jessica again uses arbitrary to express the sentiment that the specifics of the choice do not matter (line 164). This is an excellent feeling to have at the start of a problem, and all students express it at the beginning of the episode. However, at this stage in the problem, the choice is no longer arbitrary. Before, when a student or students expressed arbitrary, the TA verbally agreed but also reinforced social agreement (and to a lesser extent, consistency). Here, the TA doesn’t need to do that: the group has already strongly activated those resources.

It may be that Jessica’s continuing use of arbitrary is so strong that it masks use of consistency across an entire problem for her; another possibility is that she does not see a conflict between arbitrary and consistency at this point. A third possibility is that repeated uses of arbitrary are an effort to get the TA to state the conventional answer, thereby finessing the social agreement – and effort – that defining a coordinate system entails.\(^9\) In any case, Jessica is silent for almost three minutes while the group works on without her, emerging only when the topic is changed (line 233).

4.5.2.2 Choosing \( \hat{\theta} \) the first time

Once the group decides to use polar coordinates, the TA asserts that the two coordinates in a polar system are \( r \) and \( \theta \). The TA then leads a discussion of which directions \( \hat{r} \) and \( \hat{\theta} \) should be, starting with \( \hat{\theta} \).

\[^9\text{While the power dynamics of working in groups with a TA present are interesting, they are outside the scope of this discussion on the plasticity of coordinate system resources.}\]
Jessica and Rose quickly assert that $\hat{\theta}$ is tangent to the motion of the bob and perpendicular to the string, and the TA draws two unit vectors perpendicular to the string ($\hat{\theta}_1$ and $\hat{\theta}_2$ in Figure 4.3). The TA asks “which one gets to be $\hat{\theta}$?” Rose chooses the inward pointing option ($\hat{\theta}_2$), and Vivek and Martine profess confusion about her choice. In the excerpt below, the participants discuss their choice and its necessity.

Jessica 0:02:05 I don’t think... does it matter? I mean all it will be is a negative or positive.

Vivek 0:02:11 We’re taking this as theta, right? (indicates angle between string and equilibrium on small diagram)

TA 0:02:14 Well, I don’t know.

Jessica 0:02:17 Won’t it be negative or positive? I mean...

Rose 0:02:20 Well, where do we want to call theta zero? If theta’s zero in the center (places pencil to indicate vertical) don’t we want theta-hat to be that way (outwards pointing vector)... so that its... I guess I don’t understand what the difference would be.

TA 0:02:30 They can’t both be theta-hat.

Vivek 0:02:32 Yeah.

TA 0:02:34 ’Cause that would say that theta-hat has two different directions.

Rose 0:02:40 So... but its arbitrary though, isn’t it?

In that interaction, Rose asserts that $\hat{\theta}_1$ is the correct $\hat{\theta}$ because, if $\theta$ is zero at equilibrium, a natural choice is to have outwards be positive. This choice harkens back to directionality: away from the origin is more easily positive, a statement
about \( \theta \)'s value. As she trails off at line 56, Rose admits that she “[doesn’t] know what the difference would be” between inward and outward \( \hat{\theta} \) choices. Her statement may be an indication of confusion – would the problem be different if we used the other? – or of arbitrariness – does it matter which we pick?. In either case, her statement at line 62 is a clear indication that she is using arbitrary.

63 TA 0:02:42 Pretty much, yeah. So you have to decide, and I’m going to suggest that as a table we decide... unified, so that everybody agrees.

Jessica 0:02:52 Well, I think in this case, since its way on the end, its going to be coming this way (gestures towards vertical) so we should have it, on closer to this line.

69 Rose 0:02:57 (TA points at inward pointing vector) Yeah.. pointing back to equilibrium.

The TA suggests that the students agree on a definition for \( \theta \), an overt use of social agreement. Jessica responds that a natural choice is to use \( \hat{\theta}_2 \), because the bob is about to be moving in that direction. It seems that Jessica has connected directionality with the direction of motion, not the position of the bob, or that she is thinking about directionality instead of value. Rose agrees, contradicting her earlier intuition. Just after this clip ends, the TA and Vivek verbally agree with Rose and Jessica, again expressing social agreement and signaling the end of the segment.

At this point it seems that they have implicitly chosen the angular vector, \( \theta \), to be zero at the bob’s current position and increasing in the direction of motion, an unconventional but not uncommon choice.
4.5.2.3 Choosing ̂r

After the student discuss ̂θ, the TA asks about ̂r, and the participants quickly agree that ̂r is radially outwards along the string ( ̂r₁ in Figure 4.3). Later in the episode, the TA asks about the direction of ̂r were the bob to be to the left of equilibrium, testing the students consistency across cases. They respond that it is still radially outwards ( ̂r₂ in Figure 4.3).

The TA returns to ̂r again near the end of the episode. She asks,

TA 0:12:28 Okay, so we know that this r-hat (right of equilibrium) sort of points off that way (along string) and this r-hat (left of equilibrium) points off that way (along string). You guys didn’t disagree about the r-hats. Is it okay that they’re pointing off in different directions?

The TA’s question is explicitly about consistency (“is it okay…”) and directionality (“…pointing off in different directions?”). Vivek responds that they’re both “out of center”, and the group nods assent. His response is an indication that ̂r expresses a legitimate direction (“out of center”) in the same way that ̂x expresses a legitimate direction (“leftwards”). This refinement of directionality and the ease with which it appears speak to its solidity in the context of polar systems.

4.5.2.4 Finding zero

After choosing ̂θ and ̂r the first time, the TA asks the students where (0,0) is, in an effort to bring issues of directionality and value to the fore by considering origin. For a polar coordinate system, this is not a fair question, though it is perfectly normal to ask in a Cartesian context. Wes and Derek struggle with the issue of origin in Section 4.4.3.
Vivek responds quickly that the origin is at the pivot, but Rose replies that it would make more sense to have it at the ball’s current position. Martine replies that should be at the ball’s equilibrium position. Jessica responds that the origin should be vertically aligned with equilibrium, but horizontally level with the bob – a position somewhat above the equilibrium position. That four students grapple with this new problem and recognise origin quickly bespeaks origin’s existence as a resource for them.

Rose, Martine, and Vivek’s choices all have elements of the conventional choice. Rose seems to be arguing for body-fixed coordinates, another connection in her well-developed and well-connected arsenal of coordinate system resources. Martine’s choice, attached to the natural sense that the movement is symmetric about the equilibrium point, seems to focus on where \( \theta \) would be best zeroed. Of the three students, Vivek argues his choice most persuasively:

111 Vivek 0:04:22 (unclear) I could do this one (points at pivot), because from here you measure \( r \) (indicates length of string)

Vivek uses consistency to help him make his decision: his origin is consistent with how the participants have chosen to measure \( r \). An origin decided – though the line where \( \theta \) equals zero is still implicit – the group nods assent, an expression of social agreement. From the high angle of the video, I cannot tell whether Vivek asks for this assent nonverbally.

As the students start to copy over the derived coordinate system onto their individual papers, the students and the TA mention again the arbitrariness of their choice, the necessity of being consistent within a problem, and the social agreement norm of working in groups.
4.5.2.5 Testing Coordinates

On the surface, it appears that all students agree on a polar coordinate system, including the directions, names, and zeroes of both coordinates. To test this agreement and understanding, the TA asks about the coordinate system if the bob were to be on the other side, to the left of equilibrium. The students quickly agree that \( \mathbf{r} \) will still be radially outwards, but confusion ensues in the discussion of \( \hat{\theta} \).

Ed, Martine, and Vivek agree that \( \hat{\theta} \) should be consistent with the prior definition: pointing inwards towards equilibrium. Rose agrees that it should be consistent, but disagrees with their conclusion: she argues that, to be consistent, \( \hat{\theta} \) should continue to point the same direction, clockwise or counterclockwise.\(^\text{10}\)

165 Rose 0:06:53 So wouldn’t you want it to point this way (points at \( \hat{\theta}_1 \) in Figure 4.3) so that over here (points at right ball position), like theta’s zero in the (indicates equilibrium position), so over here theta would be positive (right of equilibrium) and over here it would be negative (left of equilibrium). So don’t we want it [\( \hat{\theta} \)] to point that way (\( \hat{\theta}_1 \))?

In her argument, Rose has two main points. First, to be consistent, \( \hat{\theta} \) should continue to be in the same (clockwise or counterclockwise) direction, no matter which side of equilibrium the bob is on, a statement of value. Second, the most easy choice is anticlockwise, so that the bob’s position is in positive \( \theta \) to the right of equilibrium and negative \( \theta \) to the right of equilibrium. Rose’s second point is at odds with the group consensus in Section 4.5.2.2, Choosing Coordinates, but it is in

\(^{10}\)It can be difficult to keep track of who thinks \( \hat{\theta} \) should be in which direction at which position, and when he or she thinks that. As this thesis is concerned with resource activations and plasticity, the details of \( \hat{\theta} \)’s direction are less important than the reasoning used to support the choices.
agreement with her original intuition about the direction for $\hat{\theta}$. A resource graph of Rose’s argument is presented in Figure 4.14.

Vivek follows Rose’s lead, and labels $\theta$ on the diagram as the angles that the string makes on either side of equilibrium. Rose continues her reasoning, restating her argument to more clearly use value:

Rose 0:07:05 Yeah, so we want the unit vector to point this way ($\hat{\theta}_1$) so that theta over here (left of equilibrium) is minus.

After an interlude where Vivek worries about the effect a negative $\theta$ would have on “the equation” and the group dissolves into everyone talking at once,$^{11}$ Rose continues her description:

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$^{11}$Sadly, I can’t make sense of Vivek’s argument, partly because he mumbles too much, and partly because other people start to talk over him, further obfuscating his words.
Rose 0:07:58 And plus... I don’t know if that has anything to do with it, but don’t you want r and... r-hat and theta-hat to have the same orientation no matter where you are?

TA 0:08:05 What do you mean?

Rose 0:08:06 Well, like, over here (right of equilibrium) if you cross them, ’cause they like kind of determine you plane hand.

TA 0:08:12 Okay.

Rose 0:08:13 If this is pointing over here (vector pointing towards equilibrium to the left of equilibrium) then you plane flips over.

TA 0:08:15 What do you mean by “plane flips over”?

Rose 0:08:17 Well, over here your cross product is (uses right hand rule)... pointing in, I think, over here (uses right hand rule) its pointing out.

It doesn’t matter, but don’t you want it to be... the same, no matter where you are?

TA 0:08:29 You want, you want your coordinate system to have the same handedness no matter where you are?

In her argument, Rose uses consistency and calculational ease in a geometric explanation to motivate and require that \( \hat{\theta} \) have a specific direction. Her argument incorporates cases to the left and right of equilibrium (using value), just as Bill looked at each case for the signs of the air resistance and spring forces in damped harmonic motion in Section 3.3. In Bill’s argument, the signs of the forces in a differential equation were at stake and he used coordinate systems to justify his response. In Rose’s argument, she uses handedness and consistency to choose among
\( \hat{\theta} \) directions in her coordinate system. *Handedness* and *consistency* are thus more solid than *direction* as applied to \( \hat{\theta} \), and well that it is so.

Rose’s continued description is a beautiful and complete argument for why \( \hat{\theta} \) should have the direction she chooses. However, it is lost on her classmates. A few seconds later, expressing their collective confusion, Vivek tells the TA that, “You need to say something. We are stuck.”

### 4.5.2.6 An analogy to Cartesian

To help the students make sense of Rose’s argument for \( \hat{\theta} \)’s invariance, the TA invokes Cartesian coordinates. She draws an x-y plane, and asks them to consider the direction of \( \hat{x} \) in each of the four quadrants. By considering each quadrant, she hoped to decouple *value* and *direction*: the value of \( x \) may be positive or negative, but the direction of \( \hat{x} \) is always in the positive \( x \) direction; \( \hat{x} \) does not switch direction when \( x \) changes sign.

Just like Bill reasoned about the force signs, Rose considers both positive and negative cases in arguing for *consistency*.

Rose 0:10:19 If x-hat was pointing this way [negative direction], then

you’d... Okay, find your value of x at this point [in 3rd quadrant]. You’d have this magnitude [value, a negative number] times this direction [x-hat, in the negative direction], which would make it [the vector]... positive, but you want it [the vector] to be negative so it [x-hat] has to point this way [in positive direction].

In her argument, Rose uses *consistency* and *value* to determine an appropriate *direction* for \( \hat{x} \), just as the TA intended (Figure 4.15). Her argument uses the same chain of reasoning that Bill uses in figuring out the sign for the air resistance force.
However, she does not conclude with an appeal for social agreement. While this could be interpreted to mean that her direction resource is more solid than Bill’s forcesign, that comparison is unwarranted: different people ask for social agreement at different rates. Lack of explicit call for social agreement is only significant when ordinarily a student asks for it (or vice versa).\footnote{The reasons for why some students ask for agreement more often than others is probably related to their self-image as scientists and mathematicians, power issues of student-student interaction and student-TA interaction (face-saving), and student’s preferred modes of speech and perceptions of their peers. In any case, further discussion in this vein is beyond the scope of this thesis.}

Martine finds the argument clear; Ed, Vivek, and Jessica are confused. Rose employs another argument – this time, arguing from the authority of mathematical convention – and the group agrees that unit vectors in Cartesian coordinates should not change direction when crossing coordinate axes. The TA then brings the idea of invariance to polar coordinates, starting with the $\hat{r}$ direction.\footnote{In the interests of clarity, some lines have been omitted; refer to Appendix B.4 for full detail}

\begin{verbatim}
TA 0:12:49 Okay. What about the orientation of r-hat and theta-hat?
Ed 0:12:53 Well, they always have to be perpendicular.
\end{verbatim}

Figure 4.15 Resource graph for Rose’s choice of $\hat{e}$. Note the relative paucity of resources here, compared with her graph for $\hat{\theta}$. 
TA 0:12:54 They always have to be perpendicular, but we have two choices for perpendicular.

Ed 0:13:03 This kind of perpendicular (away from equilibrium) or that kind (towards equilibrium).

Ed 0:13:24 (speaking over previous student) I think stick this way. Like that. (towards equilibrium on right of equilibrium)

TA 0:13:27 Like this? (towards equilibrium on right of equilibrium)

Rose 0:13:28 And it just stays that way at the end of the... so the coordinate system is fixed to the ball.

Ed 0:13:34 Yeah.

Martine 0:13:34 Yeah.

Rose 0:13:36 And it just stays there.

Ed 0:13:37 Yeah!

Ed builds on Vivek’s refinement of *directionality* to choose a direction for $\hat{\theta}$ that will be *consistent* throughout the problem. His final “Yeah!” is triumphant: Ed finally understands the coordinate system.

To test the students’ understanding a final time, the TA poses one more scenario. What if the ball were moving in the opposite direction (as opposed to having a position on the opposite side of equilibrium)? She asks, “In theta, is [the bob] [moving] in plus or minus?” (lines 341-2). After a short discussion, the students agree that the bob’s velocity is in the negative $\hat{\theta}$ direction. Finally, the students agree that the *value* of $\theta$ will be zero at equilibrium, a choice which is both *natural* and *easy*. 
The episode over, the students copy the derived coordinate system onto their individual homework papers.

4.5.2.7 Summary

In this episode, the students quickly present a choice between polar and Cartesian, discarding Cartesian because the problem statement calls for polar coordinates. Discussion then focuses on choosing direction and zeros (a statement of value) for $\hat{r}$ and $\hat{\theta}$. Students check that those definitions are consistent across the equilibrium position and for opposite bob velocity, while also discussing what being consistent would entail.

Though the students decide relatively quickly to use polar coordinates, they have long and detailed discussion about directions and zeros – two properties of each coordinate in a system – where there are conventional choices in a canonical problem. The length and detail of their discussion shows that polar coordinates are quite plastic to them. Even Rose, whose polar arguments are most facile and well-connected, switches the direction for her $\hat{\theta}$ twice, indicating that polar is not solid in this, a most natural context for it.

While their polar resources are plastic, Rose and Vivek make cogent arguments using value and direction. Because these resources are used to justify polar, they are more solid than it. In particular, Rose’s value and direction resources seem well differentiated. It’s not clear if they are initially different for the other students.

All students strongly and repeatedly use arbitrary and consistent to help them with their choice of coordinate system. The ease and readiness with which these activate speak to their solidity in this context.
4.5.3 R-forces

After the students derive their coordinate system in exhaustive detail, they start the next step in the physicist’s solution: writing Newton’s Second Law for the pendulum, and breaking the vector equation of motion into two scalar equations. A sixth student appears and the group splits into two groups. Martine and Rose join the new student at a second table, while Jessica, Ed, and Vivek stay at the recording table. Ed and Jessica’s contributions are much more substantial absent Rose’s assertive demeanor.

In this episode, the smaller group of students derive the coordinate system again, this time while finding the components of the forces in the \( \mathbf{\hat{r}} \) direction. Though their task is somewhat different – applying forces as well as deriving part of a coordinate system – they spend less time on the task this time: the second episode is only 7 minutes, compared to the first episode’s near 17 minutes. Just prior to this
episode, they have drawn a free body diagram for the ball and have started to work individually to break the forces into their $\hat{r}$ and $\hat{\theta}$ components. Because of the nature of this task, their work is more algebraic in nature and less geometrical.

To start, the TA asks, “Okay, where are we? Oh right, making [the forces] into $\hat{r}$ and $\hat{\theta}$”. Ed responds that the tension force is entirely in the $\hat{r}$ direction, but that the weight force needs to be broken into components. However, he is unsure whether to associate $\cos \theta$ with $\hat{\theta}$ or $\hat{r}$. Quickly, he rights himself and writes and expression for the sum of forces in $\hat{r}$, using $mg$ for the weight force:

$$-T\hat{r} + mg \cos \theta \hat{r}$$  \hspace{1cm} (4.6)

Satisfied, the TA moves on, asking him what he got for $\hat{\theta}$.

4.5.3.1 Jessica’s varying $\hat{r}$

Jessica interjects, asking “What is this equal to? Force?” (line 24).\footnote{Line numbers restart at the beginning of each episode. Refer to Appendix B.5 for more details. The episode happens chronologically and textually after Coordinates-1.} She has written a different expression for the sum of forces:

$$T \sin \theta \hat{\theta} + (T \cos \theta - mg) \hat{r}$$  \hspace{1cm} (4.7)

Jessica’s expression differs from Ed’s in two major respects. She includes the forces in both the $\hat{r}$ and $\hat{\theta}$ directions, where Ed uses only the $\hat{r}$ direction. More importantly, she uses a very different definition for $\hat{r}$ and $\hat{\theta}$ than Ed does, even though she was present at the derivation for Ed’s – and everyone else’s – coordinate system. Jessica’s $\hat{r}$ points vertically up ($\hat{r}_2$ in Figure 4.17), not radially outwards ($\hat{r}_3$ in Figure 4.17). Her coordinate system is more like a Cartesian one. In the clip that follows, the TA points out these differences:

TA 0:02:00 It appears that you have made your r-hat always in the down
Figure 4.17 Jessica uses two definitions for $\hat{r} : \hat{r}_1$ and $\hat{r}_2$ are similar to $\hat{y}$ in Cartesian coordinates.
Also shown is the conventional choice ($\hat{r}_3$, radially outward), which students derived in the previous episode.

Jessica 0:02:07 No.. I made my r-hat.

33 TA 0:02:11 Or always in the up direction.

Jessica 0:02:20 Have I?

35 TA 0:02:21 Yeah.

Jessica 0:02:22 Well that’s not a squared (unclear).

37 TA 0:02:26 That’s not the same as everyone else has... see by having minus mg r-hat (points to -mg on board) you’re saying that mg is always in the minus r-hat direction, meaning that down is always in the minus r-hat direction, meaning...

41 Ed 0:02:44 The r-hat would have to be up, yeah.

Jessica 0:02:47 Oh, so it should be this way then (changes signs on r-hat components written on board).

TA 0:02:50 Now your r-hat is always down.

45 Jessica 0:02:53 Well, that’s what it is.
TA 0:02:55 Is it? It looks like its sort of down at an angle over here
(points to diagram of pendulum to right of equilibrium).

Ed 0:03:00 It’s down here (points to diagram of pendulum at equilib-
rium).

Jessica 0:03:05 Oh... so you’re saying this would (unclear) (erases +mg).

TA 0:03:13 sort of... what did you guys get?

Jessica does not seem to understand the fundamental difference between her
system and Ed’s: her $\dot{r}$ seems to have come from an implicit Cartesian system.
Her statement in line 45 is extremely telling in that respect: $\dot{y}$ points either up or
down, but $\dot{r}$ only points down for one value of $\theta$. Though her statement in line 50
is unclear, the TA’s following question to the group is an indication that whatever
Jessica said, Jessica’s amendment was partly wrong (or at least ambiguous).

After the TA opens discussion out to the rest of the group, Ed and Vivek discuss
their $\dot{r}$ forces because Ed hasn’t “done the $\dot{\theta}$ [yet]” (line 52). Ed explains why he
has written his expression (Equation 4.6) the way he did:

Ed 0:03:36 Its [the first term, “-T”, is] minus because the tension is in
the opposite direction of r-hat (points to T vector on diagram)

Vivek 0:03:43 that’s the minus T-hat er.. T r-hat

Jessica 0:03:46 okay minus, but why do you get... how do you get mg for
theta?

Ed 0:03:50 well I’v got mg cosine theta ’cause mg cosine theta is in the
direction of r-hat... it’s along r-hat... and its positive because its
pointing in the direction that r-hat’s pointing

Jessica 0:04:16 (very softly)I don’t see the cosine theta
Ed’s description is an excellent consideration of the way the components of the forces interact with the directionality of his coordinate system.\textsuperscript{15} Jessica doesn’t “see” the $\cos \theta$ (“see” here in the sense of “understand”, not “visually detect”, as neither she nor Ed have explicitly written “$\cos \theta$” on the board). Jessica says, “My drawings could be bad, but I don’t see it…” (line 67). By redirecting attention to her drawing skill, and away from her skill as a mathematician or physicist, she saves face.\textsuperscript{16} TA draws out a free body diagram with the agreed upon coordinates from the previous episode (Section 4.5.2). Jessica asks again about her drawing skill:

75 Jessica 0:05:01 So I have my free body wr...? I have it right, right? My free body diagram?

The forces on Jessica’s free body diagram are correct, as the TA tells her. It appears that the root of Jessica’s confusion is in the specifics of her coordinate system, not her sketch of the forces. After having her free body diagram approved, Jessica drops out of the conversation again, not emerging until after the episode is over.

It is possible that Jessica has activated knowledge-from-authority, a resource that shields her from having to reveal (and perform) her own reasoning. In that case, it is difficult to make decisions about which resources she uses, let alone their plasticity.

4.5.3.2 Ed’s blossoming understanding

In the Sec 4.5.2 episode, Coordinates-1, Ed acted as a chorus member in the confusion choir. He didn’t speak much, and his utterances indicate that the discussion is moving too fast for him most of the time. In this episode, he blossoms. It

\textsuperscript{15}A discussion of the resources involved in vector components, vector diagrams, and free body diagrams in particular is outside the scope of this thesis.

\textsuperscript{16}While the power and authority issues in this interaction are fascinating, I touch on them only briefly here.
seems that making a polar coordinate system was difficult for him, possibly because polar is extremely plastic to him. However, once he has the coordinate system in place, he can work with it to figure out the components of each force. In terms of the RBC model for abstraction, we would say that Ed cannot recognize polar here. However, polar can be the result of a build-with action used in the next build-with. Because he can work with it, but has problems deriving it, Ed’s polar resource is quite plastic. However, because he can work with it at all, it is more solid than Jessica’s.

4.6 Themes

In Chapter 3, Plasticity, I present an extended example of a student using coordinate systems to justify forcesign. In this chapter, I delve into the structure of coordinate systems. Though Bill used it as a single resource, referring to it without exploring its internal structure, the student work in this chapter indicates that it has a rich and varied structure.

To help examine the structure of coordinate systems, I broke the resource into three subgraphs, properties, use, and case (Tables 4.1, 4.2, and 4.3). To look at how some of these constituent resources may interact, I introduced the simple pendulum as a canonical problem in mechanics. First in HHS and later in Miniviews, we asked students to explicitly form a coordinate system for the simple pendulum.

Miniview students Wes and Derek activate polar and Cartesian, and justify their use with natural and easy. In their discussions, Wes brings up the idea of span: how many coordinates are necessary to describe the motion? In polar, only θ changes, a fact Derek exploits when asserting that polar is more easy. In contrast, Wes feels that Cartesian is a more natural choice (Figures ?? (resource graph) and 4.9 (plasticity chart) ). In the 10th week, Wes persists in using an inappropriate Cartesian
system despite professed knowledge of polar coordinates, indicating that Cartesian coordinates are quite solid to him. Furthermore, he and Derek must rederive (rather than recall) the details of the polar coordinate system, indicating that polar coordinates are plastic to them. Derek’s reasoning is more facile, indicating that his polar resource is more solid than Wes’s (Figures 4.13 (resource graph), and 4.10 & 4.11 (plasticity charts) ).

In contrast to the miniview students, the HHS students quickly determine that they will use polar coordinates. However, they must repeatedly derive the value and direction for each coordinate, striving to stay consistent across the equilibrium line (and using different definitions for what consistency might entail). Because of the depth and length of their discussion, I infer that, though they say they are using polar coordinates, polar is too plastic a resource to be used effectively for most of the participants. Instead, they must build it from other pieces. As the students build polar, different students use different arguments as to the zero (a question of value and direction) for $\hat{r}$ and $\hat{\theta}$. The breadth of Rose’s arguments (Figure 4.14) indicates that her polar resource is well-connected and more solid, while the repeated use of arbitrary coupled with ambiguity between value and direction indicate that Jessica’s is extremely plastic. Because Ed can work with it, but has problems deriving it, his polar resource is quite plastic, somewhere between Rose’s and Jessica’s (Figure 4.18).

It is tempting to try to relate the plasticity of Ed’s polar resource to either Wes’s or Derek’s from the miniviews. However, as Wes and Derek do not interact with Ed, and their task is framed sufficiently differentially, comparisons are difficult. In the coordinate-determining episode, Wes and Derek frame the choice as between polar and Cartesian. Ed, Rose, Jessica, and crew quickly choose polar, but spend all their time hashing out value and direction (Figure 4.16). Even though the specific resources that Rose uses to make her arguments differ from Derek’s, the breadth
of her connections and mutable \( \hat{\theta} \) direction suggest that her polar resource is of comparable solidity to his. Jessica, with her extreme confusion about the difference between \( \hat{r} \) in polar and \( \hat{y} \) in Cartesian, bears some similarity to Wes: both of them keep artifacts of Cartesian around as they profess to use polar. However, Wes seems more facile in working with his coordinate system. This may be an artifact of other resources activated, such as knowledge-from-authority.

The plasticity of a resource can be illustrated in two ways. Plasticity of a resource is related to the strength and quantity of resources that link to it, as pictured in resource graphs. Another visualization, which omits information about linkages to other resources in favor of more direct information about the plasticity of one resource, is placing the resource on the plasticity continuum, as pictured in plasticity charts. These two visualization strategies are complementary, and I use them both in this chapter.
Chapter 5

CONCLUSIONS

In this thesis, I detail and expand upon Resource Theory, a theoretical framework which draws from the Pieces, Cognitive Science, Process/Object, Ecological Approach, and Conceptions traditions. My expansions allow Resource Theory to account for the development of resources and connect the activation and use of resources to experimental data.

To explore these expansions and their application, I present several extended examples drawn from an Intermediate Mechanics class. In every case, the richness of student reasoning requires a careful and fine-grained theoretical approach. To illustrate the expansions, I employ two visualization strategies: resource graphs and plasticity charts.

5.1 Rich reasoning for simple pendula

The simple pendulum is a touchstone problem for classical mechanics. It enjoys many variations, and appears in multiple classes. As a representative for simple harmonic oscillation, the simple pendulum presages extensive modeling in classical waves and quantum mechanics. A physicist solving for the position of the bob as a function of time quickly chooses a polar coordinate system centered at the top of the string en route to writing and solving equations of motion. For the physicist, the choice of polar coordinate system is so easy and so natural that it hardly seems worth the time to make it explicit.

By the time students reason about the simple pendulum in an intermediate mechanics class, they have already seen it in their introductory physics class. In
mathematics and physics classes, they have developed tools for choosing coordinate systems, both Cartesian and polar. When presented with this seemingly simple problem – what coordinate system to choose? – students use their rich reasoning resources to explicitly decide between polar and Cartesian. To help them choose, miniview students argue over which system is more natural or easy and whether both systems can span the bob’s motion. Along the way, they check that their reasoning is consistent and that they agree with each other and the TA. In contrast, HHS students decide quickly to use polar, but spend a long time carefully deriving the direction and value for each coordinate. Like their miniview peers, they check for consistency while noting that the choice is somewhat arbitrary: different coordinate systems are equivalent, but it is socially expedient to agree on one choice. Their need to be consistent does not mean that their coordinate systems are the same: they each propose different origins, directions, and zeros (a question of value) as they consider cases to the left and right of equilibrium.

The richness of student reasoning about the seemingly simple problem of coordinate choice requires a gentle hand from the TA to blossom: every time the TA or another student asserts that something is true by convention, discussion on that topic ceases as students accept this knowledge-from-authority.

5.2 Significant expansion of theory

If the student reasoning requires a gentle TA hand to blossom, then its analysis also requires a careful hand to understand. Exploring the depth of this reasoning requires a fine-grained theoretical approach.

To identify the resources in play, I present resource heuristics (Section 2.3.2). To describe how they develop and relate, I present the plasticity continuum and plasticity heuristics (Chapter 3, Sections 3.1 and 3.2).
The plasticity continuum blends elements of Process/Object and Cognitive Science with Resource Theory. The name evokes brain plasticity and myelination (markers of learning power and reasoning speed, respectively) and materials plasticity and solidity (with their attendant properties, deformability and stability). In the plasticity continuum, the two directions are more plastic and more solid. More solid resources are more durable and more connected to other resources. Users tend to be more committed to them because reasoning with them has been fruitful in the past. Similarly, users tend not to perform consistency checks on them any more. In contrast, more plastic resources need to be tested against the existing network more often, as user forge links between them and other resources.

The plasticity of a resource can be illustrated in two ways. Plasticity of a resource is related to the strength and quantity of resources that link to it, as pictured in resource graphs. In a resource graph, resources are pictured as circles, and the links between them are arrows. Another visualization, which omits information about other resources in favor of more direct information about the plasticity of an individual resource, places the resource on the plasticity continuum, as pictured in plasticity charts. These two visualization strategies are complementary, and I use them both in this thesis.

To illustrate how plasticity can usefully describe students’ reasoning about physics, I give an extended example in which a student reasons about the forces in damped harmonic motion. I use the heuristics to reason about the plasticity of two resources, forcesign and coordinate systems, and show that the resulting analysis bears some resemblance to an important theory in communication science, Toulmin’s argumentation structure. In contrast to the minview and HHS students in Chapter 4, the forcesign student does not explore the structure of coordinate systems, using it as a single resource.
5.3 Looking outwards: connections and applications

To identify resources at all, a cohesive Resource Theory is necessary. Resource Theory is a general knowledge-in-pieces schema theory from physics education research. It bridges cognitive science and education research to describe the phenomenology of problem solving. Resources are small, reusable pieces of thought that make up concepts and arguments. The physical context and cognitive state of the user determine which resources are available to be activated; different people have different resources about different things. Over time, resources may develop, acquiring new meanings as they activate in different situations. In this thesis, I draw together five research traditions – Cognitive psychology, the Ecological Approach, Conceptions, Pieces, and Process/Object – to make some of the details of Resource Theory more explicit. I also compare plasticity to frames, and show that the two are different and complementary.

In this thesis, I mention teaching implications only briefly, even though they are very important in applied PER. The work in this thesis can be helpful to applied PER. The plasticity continuum can help explain why students sometimes change their minds easily, and sometimes do not. Teachers can take advantage of this feature as they help students build resource graphs, encouraging some resources’ activation while discouraging others.

At the University of Maine, curriculum designers (including me) in the Mechanics Project (of which the work in this thesis is a part) and the Intuitive Quantum Physics project have already written tutorials using resources and the plasticity continuum to help students understand topics as diverse as separable forces and conservation of energy, wave interference, the quantum square well model, and operator notation (as well as others)[96]. In our work, we exploit the resource and plasticity heuristics to help students make sense of student reasoning and present topics in a way to
encourage resource refinement and solidification. For example, in the tutorial on Separating Forces, we develop resources for separability of forces and conservative forces. The distinction between conservative and separable forces is subtle, yet important. A conservative force is does zero work around a closed path; a separable one has each term in its curl identically zero for a given coordinate system. For calculational ease, it is often appropriate to choose a coordinate system in which the forces of interest are separable. In the course of the tutorial, we draw on students' existing resources for gravitational fields as both conservative (for all coordinate systems) and separable (locally, in Cartesian; globally, in polar) as we facilitate building a mathematical distinction between the conservative and separable. By attaching mathematical formalism to conservative and separable, we hope to make them more solid.

Because the plasticity of a resource is independent of its veracity, plasticity is an inappropriate tool to tell teachers “what is right.” However, listening carefully to what students are saying and how they are saying it reveals a wealth of information for teachers and researchers alike. Resources and their plasticity present a fine-grained theoretical framework; the heuristics for both suggest how to tie the theory to experiment and practice.
Chapter 6

SUGGESTIONS FOR FUTURE WORK

In any project, there are three main avenues for future work. The theory can be improved through better connections to other theories or further specification within itself. The experiment can be improved through acquisition of data in different situations with similar tools or in similar situations with different tools. The experimental results can be applied to engineer practical solutions.

In this thesis, I collect and make explicit many properties of resources. Bringing together theories from other fields, I extend Resource Theory to deal with resource development in a continuum called plasticity. I provide heuristics for identifying resources and their plasticity from transcript. However, some open questions in Resource Theory remain. How are resources selected for activation? How might resource priming be measured? At what point does a resource become primitive? If resources activate, they must also deactivate. How? Connectionism, with its parallel distributed computing metaphor, has explored these questions. Further connections forged between Connectionism and Resources might find answers.

In addition to questions internal to Resource Theory, other theories in other fields might be brought in. In particular, Conceptual Blending[3] seems to offer a mechanism for resources becoming more solid. Conceptual Blending (shortened “blending”) poses that new ideas come from the merging – blending – of older ideas. Over time, these blends may become faster and richer. Blending finds power in emergent phenomena, an attractive feature when dealing with complicated resource networks.

Cognitive Psychology and memory research offer theories, such as learned inattention[128, 129], that may help explain why students focus on certain aspects of a problem to
the detriment of others. In learned inattention studies, if two stimuli are presented
that lead to one conclusion, one stimulus overshadows the other. For example, sup-
pose the following three phase experiment. In phase one, subjects learn that if they
are presented with a square or a triangle, they are to push a square button or a trian-
gular button (respectively). To test their learning, they are presented with a variety
of monochromatic squares and triangles. In phase two, the shapes are colored. As it
turns out, if the color blue is used, it is always used with a square. Thus, seeing the
color blue means that subjects should also press the square button. In phase three,
subjects are presented with a blue circle and asked to push a button. Subjects do
significantly worse than chance in selecting the square button. Learned inattention
suggests that linking the blue color to the square button has been blocked[129].

Changes in a resource’s plasticity with time might bear some similarities to con-
ceptual change. Using plasticity language, conceptual change might be modeled as a
process of making a solid resource more plastic, rearranging its contents and linkages,
then bringing it more solid again. A rich literature on conceptual change theories
posits many mechanisms for conceptual change[112, 92, 130, 114, 98, 54, 115, 59].
Some of that literature has already been brought into harmony with Resource
Theory[90]; other parts are too ambiguous about what constitutes a concept to
compare them rigorously with Resources. Conceptual change is usually wrought on
solid resources. When a resource changes easily, it is quite plastic and may change
back, implying that conceptual change may not “stick” on plastic resources. In par-
ticular, the instructional strategy of “elicit-confront-resolve” may be inappropriate
on extremely plastic ideas.

Social theories can enhance analysis of student work, and a more rigorous dis-
course analysis might uncover more details as yet unseen.

Of course, the theoretical frame I chose is not the only one. The same data
may be analyzed through other frames to yield other results. As a trivial example,
gender researchers may want to examine the different modes of speech that male and female participants adopt. There are fascinating power dynamics between the TA and various students which I touch on only briefly. In particular, the relationship between students’ derived answers and assertions of convention seem important. The language of framing might yield interesting results about what resources are available to be activated in each situation.

To help ground my general theoretical work, I develop one resource, coordinate systems, which can be used alternately as a resource (like Bill does in the forcesign interaction), or as a graph of resources (like students do in the pendulum problem). These extended examples constitute a plausibility and existence proof of the central theoretical developments. Though they may seem exhausting, they are certainly not exhaustive. Two initial avenues for future work immediately present themselves: asking coordinate-system-centric questions in other contexts to elicit different aspects of coordinate systems, and asking the pendulum task to other students. Two populations seem especially interesting: physics graduate students, for whom polar coordinate systems should seem more natural, and introductory physics students, for whom Cartesian coordinate systems may be the only available option.

I chose to examine students’ coordinate system resources. However, the plasticity continuum is applicable to all kinds of resources. A related task might be to detail the interplay between students mathematics and physics resources as they apply their coordinate system in solving the equations of motion. Farther afield, another study could track students’ resources over many years as they develop from introductory physics students into professional physicists.

Coordinate systems are closely tied to specific representations of a problem. As an extreme example, consider the choice between position space and momentum space in quantum mechanics. As a more mundane example, consider the effects of seeing a graph of position vs. time for a simple oscillator vs. seeing a diagram of the
same oscillator. In the graph, the coordinates are position and time. In the diagram, coordinates are only positional. Extensive research\cite{51, 130, 131, 132, 133, 47} on representations and the cognitive work in switching between them suggests that applying plasticity to representational switches would be fruitful.

Researchers using resources and plasticity have started to develop curricula in intermediate mechanics, writing tutorials for generalized coordinates, Lagrangians, separability and conservative forces, and others. Other applications are possible. Promising avenues are ones in which students ideas are already plastic, or where they are asked to apply new ideas to old situations. In particular, curricula which combine physical reasoning with mathematical formalism, or physical reasoning about quantum or relativistic phenomena seem promising.

In all of these suggested avenues for further research, one thread remains clear: using plasticity to describe the development of resources allows us to ask a new and rich questions that will develop the theoretical underpinnings of PER, connect those underpinnings with experiments, and use the resulting understanding of physics learning to improve our teaching.
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Appendix A

METHODS

In any research project, the choice of theoretical framework and the choice of methods constrain each other, and are in turn constrained by the available research settings. These constraints frame a research question. In Chapters 2 and 3, I detail some constraints on theory and research question. In Section 3.5, I mention that, to find evidence of resources’ plasticity, researchers need to investigate the processes of student thought. This theoretical choice, combined with the small numbers and naturalistic setting of an Intermediate Mechanics class, requires that qualitative methods be used. I use a grounded theory approach[134] to generate data and refine theory. In this Appendix, I detail the interaction of my research setting and method.

Research setting

Intermediate mechanics is a particularly rich place to study the interplay between physics and mathematics ideas, as students often enter with a solid intuitive grasp of the physics (which may be incorrect), but have not yet applied sophisticated mathematics. At the University of Maine, intermediate mechanics is a one-semester physics course which meets for three one-hour periods each week. Generally, one of those periods is devoted to small-group work on research-based guided-inquiry tutorials[22]. The other two are lecture-based. There is no accompanying laboratory section, though physics majors are usually concurrently enrolled in a sophomore-level or junior-level lab class. There is also no accompanying discussion section, though we offered an informal “homework help session” (“HHS”) once each week in the evening.
during the years studied in this thesis. During HHS, students worked in a group to solve homework problems. A graduate student facilitated the discussion and helped students solve problems, but did not present problems and their solutions. More detail about HHS is available in Section A.2.2.

During the years studied in this thesis, the course followed a typical intermediate mechanics schedule, starting with air resistance and continuing to damped and driven harmonic motion, energy considerations, Lagrangians, and rotational motion. Typically, about half of the students in the course are concurrently enrolled in Differential Equations; the other half have already taken it.

Data collection

In PER, there are three classic methods of data collection: the written pre/post test or survey, the oral interview, and recordings of classroom interaction. In this setting, we collect data through four avenues: weekly HHS, small group short interviews (“miniviews”), classroom video of tutorial work, and written artifacts like exams and homework. For the former three avenues, we use both video data and field notes.

Written work

As part of their classwork for Intermediate Mechanics, students write homework solutions, exams, and pretests. Written work is a static presentation of ideas. It may not – and often does not – reflect the paths by which those ideas developed, nor does it usually include the richness of description that oral work includes. For these reasons, written work is not usually an effective source of data for a study which looks at the development of ideas on a fine scale. As a result, I do not use students’ written work in this thesis.
Homework Help Sessions

The homework help sessions ("HHS") provide an another opportunity for researchers to observe the processes of students developing understanding. Students discuss homework problems, and the sessions focus on cooperatively figuring out the solutions. Students do not receive credit for attending HHS, though we hope that their homework solutions (and thus homework grades) are improved as they work together with help from a graduate student TA. It is important to note that the role of the TA in the HHS is different than the role of an interviewer in an interview. The TA facilitates student reasoning and guides them to a correct and complete solution; an interviewer probes student reasoning and tries not to lead subjects to the right answer.

In an HHS, students usually sit around one table and frequently write on the table's whiteboard. We find that a high-angle camera view that captures the whiteboard is the most effective means to take data. This view does not capture students faces, but it often captures their hands. From field notes, we note that students do not often use expansive gestures that a more low-angle camera would be able to capture.

In a typical HHS, 2-4 students appear and the session generally lasts 1.5-3 hours. Most students usually show up within a few minutes of the listed start time. However, they are free to show up at any time and occasionally come late. When several students have sat down, the TA generally starts the HHS by soliciting for problems. The students name several problems from the homework. Unless popular opinion says otherwise, the TA selects the lowest numbered problem to work on first. Students work as a group, or, rarely, as two groups.

The TA then elicits from the students what work they have done so far in the problem, or where they would start. Students write their in-progress solution to
the problem at hand on the whiteboard, trading the marker around as individuals get stuck. The TA facilitates discussion and helps to draw student attention to different aspects of the problem as necessary. After every major chunk of reasoning, discussion pauses so that students can, in their own words, describe the reasoning steps they have just worked through. Because HHS are designed to help students with their homework, writing homework solutions is a legitimate activity. Some students elect to write solutions at the HHS; others write notes and clean them up later for submission.

Miniviews

Because HHS are student-led and have variable membership each week, it can be difficult to probe student understanding or track specific students over the course of a semester. To address these concerns, we run weekly short interviews with small groups of 2-4 students for the entire course. In PER, the individual interview is a more common data collection tool, even though a group interview more closely mimics natural problem solving settings. In a preliminary study, I found that students were more likely to be verbal and varied about their sense making in a group of their peers than in an individual interview. In groups, students respond to their peer’s arguments and confusions by varying their resource uses. Furthermore, a single interviewer can only hope to mimic the wide variety of arguments presented by students. In other fields, the focus group interview is a common and valid research technique[135]. The small group interviews in this study are single moderator mini focus groups. These “miniviews” are designed to last 15-20 minutes and occur directly after mechanics class. Miniviews use the same high-angle video camera as the HHS for the same reasons.
In Spring 2006, three groups of students were interviewed, one group on each of Mondays, Wednesdays, and Fridays after Mechanics class. Students were recruited at the start of the semester during Mechanics class, and grouped according to available time. Students were encouraged to sign up with a friend, if desired. Thirteen miniviews were conducted with each group as part of a larger study on the interplay between physics and mathematics ideas. In this thesis, I analyze data from two sets of miniviews concerning coordinate systems.

In a typical miniview, students gather with the interviewer around a table with a whiteboard. The interviewer presents the task, and the students discuss how to solve it. In contrast to the HHS, the interviewer probes the students’ thoughts but does not attempt to guide the students to the correct answer. Tasks loosely follow class activities, but unlike homework problems, are not directly related and students are not graded on their participation or answers. After about 20 minutes, if the students have not solved the task, the interviewer thanks them for their time and ends the miniview. If students solve the task early, the interviewer may probe their reasoning more deeply, extend the task, or end the miniview early. Most miniview tasks were completed on time.

Two miniviews in Spring 2006 related explicitly to coordinate systems and their use in the simple pendulum problem. They occurred in weeks 4 and 10, and followed nearly identical protocols. In the pendulum miniview, three miniview groups were asked about the motion of a simple pendulum. To begin, the interviewer drew a pendulum attached to a ceiling displaced about thirty degrees from equilibrium. The interviewer then sketched the two forces on the bob, tension and weight, and asked the students to solve for the position as a function of time. As a first task, the interviewer asked students about the details of their coordinate system. A second task asked students to consider the effects on their chosen coordinate system if the
ball were to be on the other side of equilibrium. A full text of the miniview protocol follows. The typography of the protocol has been modified from the original version.

**Pendulum miniview protocol**

Draw a pendulum attached to the ceiling, displaced to the right from equilibrium. If you sit across from students, draw it in their perspective. (So that up is away from the students). So as to minimize geometry issues, draw the pendulum at about 30 degrees from vertical – not 45.

Introduce the drawing, saying something like, “This is a sketch of a ball on a string tied to the ceiling. I’ve pulled the ball over this way. When I let go, the ball will swing back and forth.”

Draw a free body diagram for the ball just after release. Don’t get hung up over relative magnitudes of forces or labeling. There should be two forces: tension radially up and weight vertically down. Draw your FBD on the ball.

Introduce the FBD, saying something like: “I’ve drawn the forces that act on the ball the instant I let go. There are two of them: tension and weight. There’s no air resistance for this problem.”

Do not use the phrases “radially upwards”, “radially inwards”, or “vertically downwards”.

Task 1: “Let’s solve for position as a function of time. ”

First question: “Sketch what coordinate system you are using.”

Some things to watch out for:

- Coordinate systems must have two normal axes: at least one force must be resolved.

- If the student chooses cartesian,
– ask about the direction of the acceleration at the instant shown. Ask what the direction of acceleration might be a little while later (closer to, but not past, equilibrium) Ask them to express those directions in terms of their coordinate system.

– If the student does not abandon cartesian after acceleration discussion, mention polar coordinates as another option. Do not draw the coordinate system; instead, mention r and theta instead of x and y.

• If the student chooses polar,

– but selects r-hat radially inwards, mention that the convention is r-hat radially outwards.

– theta-hat is of special interest. Prompt students to be explicit about:

  * Where is theta = 0?
  * Which direction does theta increase in?

Task 2: “What happens to your coordinate system when the ball is on the other side?” You may not get to task 2 in the time allotted.

Coding

To describe the events at HHS, miniviews, and interviews, I use a grounded theory approach to develop a coding scheme.

The coding scheme has two kinds of content codes, states and activities.¹ A state describes the tenor and content of discussion at a point in time. There are two levels of states, gross states and substates. Gross states refers to the length of time in these states, usually not less than two minutes, and the specificity of the states:

¹There are also non-content codes, such as participant codes which notate who is speaking.
a discussion of technical norms is not a gross state, but a discussion of sense-making is. Substates are more specific and give a finer level of detail than gross states.

Within each gross state, a number of substates further specify the discussion. While the list of gross states is meant to be a complete list of the observed gross states in HHS and Miniviews, substates are not an exhaustive account of observations in each gross state. While I have categorized substates and gross states in hierarchical way, it is important to note that substate codes are not exclusive to each gross state. For example, the social norming code of agreement occurs frequently in sense making and technical norming states.

States

Available gross states and their typical topics or modes, in ascending order of likely interest for this study are: writing time, gossip, trivial math, social norming, sense-making. As the largest category, sense-making has three major states: technical norming, metacognitive, and pattern-matching. The gross states are sorted in ascending order of likely relevance to this study.

Writing time

Where students write down the things we just figured out. Writing time has few utterances, and zero of high-quality. Writing time often occurs at the end of the problem or at the end of a significant sub problem. Following writing time, the white board is usually wholly or mostly erased. Writing time is focused on student papers, which are not readable on the video, and therefore writing time is not useful for this study.

Gossip
When the students goof off. Gossip time is not about either physics or math, and therefore not useful to the study of physics or math understanding. While it would be possible to divide the gossip into substates about the kinds of gossip, we have not done so.

**Trivial math**

Where students do some math that is time-consuming, but intellectually trivial, e.g. mountains of algebra. Different than writing time in that the writing is not summative, and different from sense-making in that they perform typical or trivial operations and do not make sense of each step. Trivial math is usually solid; non-trivial math is more plastic. What may seem trivial to the person holding the marker may be non-trivial to other participants.

**Social norming**

Where students express or derive social norms, such as which problem to work on or who should hold the marker. Social norming occurs at the start of the HHS and also when a new person appears. Social norms are important because they determine the types of allowed behaviors in a group setting. In terms of gross states, two kinds of social norming are problem management and crowd control.

In problem management norms, students discuss which problem to work on or whether they are done with the current problem. These norms are often expressed at the start and end of each episode, and are useful for episode delineation.

In crowd control norms, students express opinions about who controls the marker (“I already wrote today, so it’s your turn”), about the purpose of HHS, or about how to work in groups in general without reference to a problem. For example,
one norm, *social agreement*, refers to a sense that all persons working on a problem should agree with the reasoning thus far, before the group moves on.

**Sense making**

Where students actively make sense of a situation or problem. Common activities in this category include: frequent checks (does this makes sense?), appeals to prior work or experience (this is like that) (may be implicit or explicit; may be from earlier in HHS or from class or from previous to class), consistency checks, and completeness checks. Many people participate, and the focus changes often from person to person. Periods of high productivity are followed by periods of writing time. All of these activities have implications for the plasticity continuum, so these clips are likely to be useful for the study. Sense making is by far the largest category, and this is where most of the interesting clips come from. Within the large sense making state, three major substates occur: technical norming, metacognitive, and pattern matching.

In technical norming, students may ask or answer questions like, “What definition are we using?” or “What coordinate system are we using?” Technical norming often occurs at the start of the problem. During a problem, sometimes students renorm as they rederive or explicitly recall norms. Derived norms are more interesting than asserted ones because they better show connections in students’ minds and because they are more likely to be more plastic.

In a metacognitive state, students reflect on their own understanding or lack thereof. This state often occurs at the start of an episode, when students mention what they don’t understand, or at the finish, when they reflect on what they’ve just done.
A third subset of sense making is pattern matching, where students match surface or trivial features in a problem. If pattern matching is a short-term endeavor, it can be a fruitful means to start problems or restart stalled problems. However, if the pattern matching continues for a long time, it may indicate that students are overloaded or overwhelmed. In this case, low quality engagement means low quality data.

**Activities**

Gross states and substates refer to the tenor of the discussion, but not necessarily its content. For example, the same math steps may appear as sense-making for one group, as they figure out what to do, and as trivial math for another group, as they simply rattle off the answer. Similarly, two different activities may be both coded into one gross state: both choosing a coordinate system and naming a constant of integration could be technical norms.

Obviously, activities and gross states are not completely independent. Most activities greatly favor one or two states over others. While gossip and technical norming have no activities in common, for example, technical norming and sense-making share activities often, as do social norming and sense-making.

This list is not meant to be exhaustive, and it focuses on activities that occur repeatedly in the course during sense making.

*Coordinate systems* For dealing with coordinate systems; possible subactivites include choosing, using, and reversing a given coordinate system. Often used in conjunction with metacognitive codes dealing with arbitrariness.

*Differential equations* For dealing with how to solve differential equations. Usually via separation of variables. Also used for setting up said equations or solving the resulting integrals.
As a counter-example to these primarily sense-making activities, consider the following activities which rarely occur during sense making:

*Attractiveness* When gossiping, students occasionally discuss how attractive other people are. These people may or may not be related to the course.

*Workload* When gossiping or social norming, students discuss their workloads, exams for other classes, or expectations for this class.

Though an analysis of students’ expectations for coursework may be fascinating, in this thesis, I focus on students’ use of coordinate systems in sense making gross states.

**Data reduction**

The HHS produce hours of video each week, but not all of those hours are fruitful for research in the Mechanics Project. Due to technical problems, sometimes entire HHS are lost, or the latter half of an HHS may not be recorded. Using gross states, we divide the remaining HHS video into episodes and select some episodes for further study. Episodes which feature primarily writing time, gossip, or trivial math are left on the cutting room floor.

Sense making and (to a lesser extent) social norming HHS episodes are then screened for activity. Students’ problems and solutions in HHS, combined with research questions in the Mechanics Project, were used as inspiration for miniview protocols. Episodes which feature coordinate systems (and clear audio) were transcribed for this study.

Once the episodes are transcribed, they are coded for resource use using the resource and plasticity heuristics detailed earlier in this thesis (Sections 2.3.2 and 3.2, respectively).
Appendix B

TRANSCRIPTS

This appendix has raw transcript for the episodes detailed in the thesis. Note that the line numbering is continuous from the start of each section, and that line numbers in the thesis are consistent with line numbers in this appendix.

Forcesign

Bill 0:00:00 Sum of the forces... Why is this negative kx minus cv (points to that part of the equation written by the other student) in the y-hat added to the (unclear) both of these are being negative all the time.

TA 0:00:15 Okay.

Bill 0:00:16 Um... and I kind of have am a bit concerned so does it go over?

TA 0:00:20 Sure. Well, tell me what you get so far.
Bill 0:00:22 Well, I know that when the force is like if you have the spring at... if you use equilibrium (draws equilibrium line) that's to this side (draws trajectory on right of line) the spring is going to be pointing that way (draws arrow pointing left) and the air resistance should be pointing that way (points right), so the air resistance is going to go that way (points left and then draws arrow in that direction), so it seems like it is negative on both sides of this, like its always going to oppose the motion. So it's just that since x is negative here (points at -kx), since these change (points at left side of diagram), so the signs of these change too (points at both -kx and -cv). Positive, so it ends up being (points at diagram)... I have to start over. Acceleration is...

TA 0:00:51 So, what's going on on this (right) side of the picture?

Bill 0:00:53 The... the force of the air resistance, that's f sub a (labels top arrow), on whatever, and the force of the spring on this side (labels bottom arrow), I was going to go like, like, no changes. Once the air changes though... (erases top arrow and label)

Gina 0:01:12 There's four different (unclear)

Bill 0:01:14 Yeah, that. Because its going to be the air resistance going this way and this way... (draws arrows in both directions)

TA 0:01:17 Well, what's the difference?
Bill 0:01:19 Depending on which way you're going... Well, if you're going 
that way (points right) then the air resistance is going to be pushing 
that way (points left), if you're going this way (points left) then the 
air res... Like, [the professor for this class], he did this thing where 
you go like this (makes hand motions)

Gina 0:01:26 Yeah... (laughter)

Bill 0:01:27 If you actually do it. He did this thing with his thumbs 
where you can actually model the... Like if it crosses the thing like 
the spring force goes that way and the air force goes this way and 
then they, they hit the thing so then that the spring force changes 
and the air force changes and it goes back. (makes hand motions to 
demonstrate as he's speaking, thumb representing air resistance is 
in wrong direction)

TA 0:01:41 Is that what it looks like?

Gina 0:01:42 Yeah.

Bill 0:01:42 Yeah.

TA 0:01:42 Did you get your thumb positions right?

Bill 0:01:44 I think so.

TA 0:01:46 Okay.

Bill 0:01:47 So, I just, it seems like they can both have the same sign, 
sign... and I don’t know. This one’s position and the other one’s 
velocity and...

TA 0:01:55 Let’s worry about one of them, one of them at a time. Which 
one would you like to worry about first?
Bill 0:01:59 I don’t know, just... I’ll say velocity (points at -cv). I’d just like to prove to myself that it’s correct for every case and I don’t know, if that’s (unclear)

Gina 0:02:09 Oh, you want me to do it?

Bill 0:02:11 No, I just don’t want to slow you down, cause your seem kind of like you already know it and (unclear) (erases earlier diagram)

TA 0:02:23 So if we think about the cv term, why is it minus cv?

Bill 0:02:27 cv... (writes -cv) cv... um... before...(draws equilibrium line and base) this velocity is going to be this way (draws arrow pointing right) and the air resistance is going to be that way (draws arrow pointing left). Alright, so the velocity is positive and the force is negative and when the velocity becomes negative (draws arrow pointing left) the force is positive (draws arrow pointing right) so it changes the sign around. Of this (points at -cv). In here. Alright?

Is that right? If this is the...

TA 0:02:56 Gina, what do you think?

Gina 0:02:59 Um... I’m... isn’t... I mean...

TA 0:03:04 Did you follow that?

Gina 0:03:07 I stayed with you until you started going this way. (hand gesture off screen)
Bill 0:03:09 Well... on the outside. When the velocity goes this way (points right) the air resistance is going to be that way (points left), so it’s going to be positive velocity and a negative force, so it’s positive this and the force should be negative. Where as when the velocity is this way (points left) it’s going to be a negative v which is going to cancel that and be a positive, which is this way (points right).

Gina 0:03:30 Yes. It sort of ends up being in the positive in the x-direction.

Bill 0:03:33 Okay. Okay.

TA 0:03:37 So cv is always minus cv?

Bill 0:03:41 What? Yes. (erases -cv)

Gina 0:03:42 For the particular coordinate system.

Bill 0:03:47 Well, for how we’ve chosen to do this.

TA 0:03:49 What if positive were the other way?

Bill 0:03:52 Positive were the other way? This is positive? (indicates left side of diagram)

TA 0:03:57 (affirmative hum)

Bill 0:03:59 Then...

Gina 0:04:00 It would still be minus cv.

Bill 0:04:02 The force is that... Yeah, it doesn’t really matter, you’d just be drawing things backwards.

Gina 0:04:05 (unclear) can we not... deal with that... deal with that right now?
Bill 0:04:12 Velocity is that way (points right), the force is that way (points left). The velocity would be negative the force would be positive. Um... You just took a... Yeah...

TA 0:04:23 Okay.

Bill 0:04:25 Yeah.

TA 0:04:26 Does that... Does that satisfy the...

Bill 0:04:28 (unclear) negative kx. Um... Yeah. Okay, I get it.

TA 0:04:38 That’s the cv. Why is the kx one always negative?

Bill 0:04:41 Um... When this is positive (indicates right side of diagram) it’s always... it’s always positive on this side (right) and the force is always negative, as it should be. So it should always be negative since k is positive and x is positive. And then when it’s on this side (left) the force is always going to be in the positive direction and then... this is the (unclear). Force... Okay.

TA 0:05:07 No, wait, wait, wait. Now I’m confused.

Bill 0:05:10 No. I get it.

TA 0:05:10 Ah... I’m confused, so explain it to me.

Bill 0:05:14 Um... When it’s on this side (right), positive is this way (right). Um... So that negative kx (writes -kx on board) you end up with when it’s on this side (right) for any point the force is always going to be acting this way (draws arrow pointing left), and that’s a negative force. And since k is positive and x is positive on this side, the force should always be negative. And over on this side for any point the force is always going to be acting this way (draws arrow pointing right)... the spring...
TA 0:05:48 Okay.

Bill 0:05:54 Right. (erases diagram)

TA 0:05:54 Gina had an idea...(clip ends)

**Week 4 Miniview**

TA: So, I want you to take a look at this. This is a pendulum.

1 Wes: mmhmm. Ok. I thought you were going to draw a discontinuity in a graph and I was like 'aw, come on.'

3 TA: No, nono. We’re not that cruel.

Wes: yeah.

5 TA: Whoa, that’s a totally backwards M. Um. Well, that’ll learn me...

Wes: Well, backwards to who?

7 00:37

TA: To the world. So, two forces on a single pendulum.

9 Wes: Mmhmm. More or less.

TA: The tension force, and the force due to gravity.

11 Wes: We get to neglect air resistance.

TA: We get to neglect air resistance.

13 Wes: That’s crazy.
TA: Thank goodness. Ok, umm, so what we want to talk about today, we’re not going to find an equation for the position of the system, with respect to time. But we’re going to start along the path of finding an equation for the position with respect to time.

1:01

Wes: We’re not going to find one?

TA: We’re probably will run out of time before we find one.

Wes: Really?

TA: Yeah.

Wes: Kay.

TA: Yeah, ’cause we want to derive it, we don’t just want to write it down and say, this is it.

Wes: Well, I know, I don’t want want to derive it. But I’ve seen it derived.

Derek: We’ll if you’ve seen it derived, you’re more than welcome to do it.

TA: So, so, where would we start, along that path, guys?

Wes: [inaudible]...We could start with like, Newton’s stuff, I guess.

TA: (hands Wes the marker)

Wes: Thanks.

TA: Show me what you mean.

1:37

Wes: Uh huh. (long pause)...um.
Derek: Start with Newton’s Second Law. It’s two points on every question.

Wes: MmKay.

TA: You can write what’s ‘up’ for you. You don’t have to write ‘up’ for the camera. This is [oriented] just so you both could see it.

Wes: Okay. This is consistent.

Derek: Sure.

TA: Ok.

Wes: Yeah, well, mmKay. This squiggly over here (draws the sigma in front of F). (pause) um. That points...can’t we just use energy?

(laughs)

TA: No, energy’s not going to tell you the position with respect to time.

Derek: That’s ok, it gives us the energy of the system.

TA: It does, it does. But that doesn’t help you if you want to know where it is at a given time.

(pause)

Wes: Why not?

Derek: Because it just won’t. Energy doesn’t work that way.

2:45

(laughs)

Derek: you can tell how far it’s moved, I think. I think you could do that, but I don’t think you could find position.

Wes: So it has some sort of mass, obviously.

Derek: Yeah.
Wes: It has an arc. (pause) arc length. (pause) vs. time. (pause)

3:10

(Derek laughs)

Wes: What?

Derek: Kay.

Wes: I'm just going through stuff in my head.

Derek: Alright.

TA: No, that's good.

Derek: That's fine.

(pause)

TA: So you wrote down Newton's Second Law, here.

Wes: Yes.

TA: What would be the next step in applying that?

Derek: Figuring out what all the forces are. And I think that's what he's trying to do.

TA: Mmkay.

3:33

Wes: Yeah, so? Sure. We'll do that. (pause) At what point?

Derek: At any given point.

Wes: They're different angles.

Derek: Tension is still tension. (pause) What are the tangential forces?

Wes: So sum of forces at 'time start'?

Derek: At any given time it's still going to have a tension force.

3:59
Wes: Well, at any given time, you have a theta and an \( F_g \).

Derek: I didn’t say they were equal! I said that they are forces.

Wes: I know! (draws) that’s at any given point.

Derek: Mmhmm.

TA: Mmkay.

Wes: Mmkay. Well if you want to sum forces at it like you’re going straight down as opposed to...

(pause)

Derek: The force is still going to be the force, no matter what. (pause) And...since there’s only two forces acting on it, we can assume that should always equal F of g.

TA: Why is that?

Derek: No, I’m sorry.

4:45

TA: Why’s it wrong?

Wes: Is it wrong? Is it really wrong?

Derek: Yes, it’s wrong. ’Cause there’s acceleration.

(Pause while Wes draws)

Derek: Yeah, anybody can draw a line that makes it look longer.

Wes: No, all I was saying is \( F_g \) is over here, it’s not over there. Which, just, is obviously what you said.

5:10

Derek: Yeah.

Wes: BUBBYE! (Erases)
Derek: No, there’s acceleration, therefore it has to...

Wes: What’s the letter for the period?

TA and Derek: T.

Wes: Little t or big T?

Derek: Does it make a difference?

TA: Big T, usually, by convention.

Wes: Just...

Derek: Sure. But we’ve already defined big T as being the tangential...

Wes: So what?...well, we’ll just add 'force'. (writes) Kay?

TA: Mmkay.

Wes: Okay.

Derek: Uhh

5:45

TA: So, let’s look back at, at Newton’s Second Law, here. It sounds to me like, like you think this is the way to go, but you’re having trouble figuring out how to add the forces.

Derek: What he’s trying to do is figure out how to express the tangential force.

TA: What coordinate system are you using?

6:07

(Pause)

TA: Let’s make one.

Wes: I’m not using any.

TA: Do you need to have a coordinate system?
Wes: No.

Derek: She’s asking, therefore we probably do. Remember, this is their whole trick to try to get you to second guess yourself. That’s all these tutorials are.

Wes: (points) He loves this. We may not use it. (Draws cartesian coordinates) Jeezum crow, that’s an x.

6:34

Derek: Out of curiosity, why not use polar?

Wes: Eh?

Derek: Why not use polar? We’re dealing with angles that are changing?

Wes: We’re going to deal with T, we’re going to deal with this, L.

Derek: Right, that’s all constant, though.

Wes: Yes, and those are the only things in it. There’s not even mass in it.

Derek: Mass…technically…[inaudible].

Wes: Yeah.

Derek: No it won’t.

7:00

TA: I, I’m, mmkay.

Derek: If we use a polar coordinate system, all the r…all the radii from the various points, are just going to be constant. The only thing that’s ever going to change in this is angle.

Wes: Radii? Yes. [inaudible].

Derek: Makes the math easier.
Wes: I’ll use the eraser, so my hands don’t get black. (erases). (draws polar coordinates) Here’s theta, this is an r. Does it really matter?

Derek: No.

Wes: Good. Umm...We can define this though, right? Length?

TA: Yeah, you can call that L, that’s ok.

7:44

Wes: That’s good. So you would want an equation that is any position in terms of time.

TA: Well...the position of the...yeah.

Wes: The position of this–

TA: In terms of time, yeah.

Wes: Now, do you want position in terms of this way (gestures horizontally on the diagram)? Or this...

Derek: Do you want like x,y position?

TA: All, all I want to be able to do is tell you where it is–

Derek: It’s much easier to use angles.

TA: –at any time.

Wes: Yeah.

Derek: Hence, why it’s easier to use angles.

Wes: To just know where it is. Totally.

TA: Yeah.

Wes: Mmkay.

8:19

Wes: What’s the...(pause)
TA: Tell me more about this coordinate system you, you set up here.

Wes: Just polar.

TA: Kay... What direction is the-

Wes: Positive.

TA: In this picture? Yeah, which way is it.

Wes: This is positive theta. Counterclockwise.

TA: Which way is this, r.

Wes: r is (gestures to the pendulum string on the diagram). If-what?

Derek: It really doesn’t make a difference how you define it.

TA: I–

Wes: Well–

TA: I’m just curious–

Derek: I’m just saying.

TA: How you defined it.

Wes: Or-i-gin (writes).

TA: Okay.

Wes: And there’s that angle there.

TA: Sure.

Derek: Sure, works for me.

8:58

(pause)

Derek: As long as it’s defined.

TA: So, which way is r hat?
(pause)

Wes: Opposite t hat? T sub F hat?

TA: Mmkay, so...r hat is pointing out?

Derek: Yeah.

TA: Okay.

Derek: Points away from this surface.

TA: Okay.

Wes: [inaudible, includes the word surface].

Derek: She didn’t define it being a wall or whatever. Which means this could be a magnetic situation.

(pause)

9:36

Derek: [inaudible, includes the word puppy].

(pause)

TA: Okay. Which way is, which way is theta hat in this situation? I see a theta.

Wes: Mmhmm.

TA: Positive theta.

Wes: It’s counterclockwise. Yeah, counterclockwise. (pause) umm.

10:00

Wes: Now, me just thinking about this. The only connections I’ve made so far is, if I wanted to find a ratio between... (long pause. To 10:45) Um, time and, we say it started at some angle, right?

TA: Sure.
Wes: It’s just like a, yeah, it’s gonna be some sort of sinusoidal pattern. I would need, eventually, a ratio between... this length here...and...so you know i’m just trying to think what it would depend on. I mean, if we’re neglecting air resistance and mass wouldn’t matter.

Derek: Mass is going to make a difference.

11:29

Wes: It is?

Derek: Yeah. (Pause) [PHY] 121. Root mg over L.

Wes: For what?

Derek: For, ah, period.

Wes: Really?

Derek: Yeah.

Wes: That’s T (writes the formula on the board)

Derek: That’s probably wrong, I just remembered that equation.

12:00

Wes: Is it root–

TA: How can we check to see, is there a quick check we could do that would tell us if that would be reasonable or not?

(pause)

Derek: Actually it might...let me see that for a second. (grabs marker).

We do–

TA: We’re kind of getting down–definitely off topic, so I’m going to pull you guys back in. Talk about this coordinate system some more, cause i think....I’m just a little confused. Could you draw me like a little–
Wes: Does it even matter?

TA: I don’t know. Does it matter?

Derek: See, it’s this entire process to get us to second guess ourselves.

TA: I’m just curious about how you guys have a, have it set up here.

Wes: I would say if we had it in the traditional x,y, I just don’t know where to go from...the start point.

TA: Okay. (long pause). How about if we have it polar, do you you know where to...

13:00

Wes: I would just break it down into friggin, um, arc lengths. Kinematics equations with forces and do some trig.

Derek: So do it.

Wes: And work out all the math. Hey, I don’t want to work out all the math. Goddammit (grabs marker). Well, let’s say this is L and say this is here, and this is a triangle, and that’s a right angle, and that’s some other distance I’m not going to name, yet, cause I don’t feel like it. But...this is our theta up here. The sine theta...(writes). Yes, no?

Derek: Mmhmm.

Wes: Okay.

13:53

Wes: So we’ve got these distances. And...

Derek: [inaudible]...L is a constant. That’s fine. [inaudible]
Wes: Mmkay. And, uh, we say our starting height (pause) is what-
277 ever distance, L minus this. That’s our starting displacement from
279 wherever it’s going to be at the bottom. Then I’d use energy and
281 figure out what its speed would be at the bottom. But... we’ll stay
283 away from energy, I guess. (pause) Cause my mind’s trying to be
285 connected to...we want something to tell its position with respect
287 to time. (pause. 15:00) And if you want position in x,y, then, it’s
289 gonna be...stupid to do. To try. It would be easier to do position
291 as opposed to angle of displacement. (hands marker to Derek, who
293 begins writing)

TA: Does uh–

Wes: Rest point.

TA: Does a position based on angle and how far you are away give you
289 the same information as it’s x,y coordinates.

Wes: Well I can’t think of any way to put it in terms of– you want what’s
291 this point. What are it’s coordinates at this point?

TA: 'Where is it?' 'Where is it?'

Wes: But you’d have to do that in terms of coordinates, right?

TA: Sure.

Wes: Sure.

TA: That’s–

Wes: And, uh...so I would say I don’t know any functions that would
give you out two parameters.

TA: Mmkay.
Wes: At the same time. [inaudible] I mean, you could base it on y and say ok and then plug it into a different equation and get your x.

TA: Okay.

16:00

Wes: If you want a specific point, I would almost want to take the route of what’s it’s angle of displacement. But,

TA: Would displacement tell you exactly where it was? At any time?

Wes: Huh? Well, no, you’d have to figure that out. But,

TA: Wh- are you? Sorry, you’re doing analysis over here, are you finding you have...

Derek: That’s wrong (gesturing to his writing).

TA: That’s wrong, ok. Does it give you out seconds?

Derek: If you manipulate a few things, it will do it. Ah, it’s Lm over g.

Wes: Lm over g?

Derek: Yeah, in order to get time.

Wes: Really?

Derek: Yeah.

Wes: Well, ok. Lg over m.

Derek: Sorry. That was going to irritate me until I figure it out, so I had to.

TA: Alright.

Wes: Do you know of any equations that will answer the questions of where exactly it will be with respect to time.

TA: So, we’re kind of talking about—
Derek: Oh, no. I’m listening to you guys, too.

TA: So, I haven’t heard your wording. So just knowing the angle, is that enough to tell us where it is, at any given time? What do you think?

Derek: Well, if he knows the angle and it’s based on polar coordinates, yes, he’ll know where it is at any given angle. (pause) What?

TA: What do you think? He--you were saying...

Wes: I was only saying, if you want, cause you were questioning the coordinate system, it’s ok to go back to x and y, but to give out an answer that’s going to give a precise exact location in the x and y, you’re not going to get a function that gives you both components at the same time. You can get one and plug it into a different equation for the other.

17:39

Derek: Right. That’s why it’s easier just to--

Wes: But in polar, if you want it’s exact location, in space, you have to have it in reference to something.

Derek: The origin.

Wes: So if you have it referenced to the origin, you know that this length is going to be constant, which is r in this system, and so you could say it’s anywhere there.

Derek: But if we consider this to be zero degrees.

Wes: Yeah

Derek: Then you can figure it out.

18:01
Wes: That’s about as exact as you’re going to get from one single equation.

Derek: Yeah.

(pause)

Wes: So I was trying to work backwards in my head, what the goal is, and work back, okay, based on that, what type of parameters are you going to have in your function, so.

TA: So I think we’re about out of time.

18:30

Week 10 Miniview

Wes: You know, I hate this problem.

1 TA: I’m sorry.

Wes: No, really.

3 TA: Okay. So, what’s the first step, here.

Derek: To erase it and go home.

5 Wes: The first step? (Pause) Position on what axis, with respect to time?

7 TA: Any way that you can describe the position.

Wes: Kay, cause I tried it with radians and I got a non-integral–non-integrate-able integral. And then I stopped doing, put it down, and decided I never wanted to see this pendulum question again. So now that I see it again, I kind of almost don’t want to do it at all.
TA: So you said you tried it using–

Wes: I tried it doing, first, the energies.

TA: Okay. I'll tell you that way, that’s not the way to go.

Derek: No, it really isn’t.

Wes: I stopped halfway doing it with, uh, I can’t remember.

Derek: It’s periodic motion, therefore we can assume a sinusoidal or e function. Oh, no. Not e function.

Wes: Ok, so let’s pretend it’s a sine function.

TA: So, first, first thing to do–

Wes: Like phase shift and stuff.

TA:–setting up a problem from scratch, would be to define a coordinate system.

(pause)

Wes begins writing)

Derek: Sure.

Derek: Well, how are you defining one?

Wes: I don’t know...doesn’t matter cause i’m going to transpose it...

Derek: Well,

TA: x and x.


TA: Oh. Okay. What do you think about this coordinate system?
Derek: I don’t like it. Why don’t we, ah, set the coordinate system at...

Wes: To what?
Derek: So that–
Wes: Polar?
Derek: –the origin is at the point of the hinge? And so that one axis
points at one maximum..direction and this axis points in the other
one. If we have to, we can define our own coordinate system. We
have the math to do that. It’s a pain in the ass, I’ll admit. But we
can do it.

Wes: At one maximum? So then you’re going to say at our initial problem
is 90 degrees from maximum to maximum.
Derek: Even if it isn’t, you can redefine your, ah, coordinate system–
Wes: So x and y are not–
TA: So you’re going to make a coordinate system where the–
Wes: –not independent
TA:–axes aren’t orthogonal.
Derek: No, they are orthogonal. You can–
Wes: Would you give it a second?
Derek: No, I don’t listen to you, you know that.
Wes: I know. I’m saying if these maxima are not ninety degrees, then
they’re not orthogonal.
Derek: Right, you can define your own system, ah, coordinate system so
that your x can go like that and your y can go like that (draws an
acute angle).

TA: Okay. We could do that.

Derek: It’s calc three. You did do this, yes?

Wes: More math. Yeah, I think we’re taking this problem a little over-
board. It should be–

3:08

Derek: Oh, I know we are. But...

Wes: This is the problem. I hate myself. Anyway. Okay, I’m back.

TA: Alright. So let’s pick a coordinate system.

(pause)

Derek: Polar.

(Wes laughs)

TA: How would you define that here?

Derek: You have a given r that’s constant.

Wes: Zero. R equals. What’s the one with the squiggly line? Theta!
equals zero.

TA: So. Draw.

Wes: (Writing) Theta equals zero.

TA: Kay.

Wes: This way. Our origin.

TA: Okay.

(pause) 3:57
TA: Which direction is positive theta?

Wes: Huh? uh. (writes)

Derek: Why don’t we define...that works.

TA: Okay. (pause) So, what’s the next step?

(long pause)

4:23

Wes: I don’t like these whole step by step things.

Derek: So go ahead and just solve it, then.

TA: Yeah. When I say ‘what’s the next step’, I mean proceed in the

solution of the problem.

Wes: Why would we define the coordinate system...in fact, I hate polar.

Derek: Fine! Define it in cartesian.

Wes: I wouldn’t define it in either, it’s irrelevant.

TA: Okay. So,

Derek: So what are your variables.

Wes: I’d just write...I’d just write a sine equation.

Derek: How do you know what accurately defines it? How do you know

it goes between ninety and negative ninety?

Wes: It doesn’t, necessarily have to.

5:01

TA: So what would you do if you didn’t know the answer was sinusoidal?

The question was prove from first principles—

Wes: That—
TA: --what the position with respect to time was. If your first step wasn’t
to define a coordinate system. What would you do first?

(pause)

5:30

Wes: Maybe I’d draw a free body diagram.

TA: Okay. Sounds good. Let’s draw a free body diagram.

Wes: (writing) That’s an e, that’s an f.

TA: Okay.

Wes: I’ll take this as our starting position, right now.

TA: Okay.

Wes: (writing) Tension of string. I’ll don’t want to put r for rod.

TA: There’s an s.

Derek: Tension is constant and e is constant.

Wes: Anything else? Wind resistance maybe?

TA: No

Wes: Damping?

TA: No no.

Derek: The way she says no to that makes me a little afraid.

TA: Let’s keep it simple.

Wes: Magnet field this way? Aluminum ball?

TA: No.

6:20

Wes: Okay, free body diagram.

TA: Okay.
Wes: That’s and F B D.

TA: Okay.

Wes: Umm. Kay. Now maybe we’ll write an equation with the sum of the forces.

TA: Alright, sounds good. Are you happy with?

Derek: Sure. I'm not sure why he's giving me these smartass looks.

Wes: I have to! I have to bring you into the conversation somehow. So whether you’re yelling at me or not. (Derek laughs 7:00). Okay.

These no longer exist. That’s our coordinate system. That’s the origin.

TA: So, so now you want to choose a coordinate system.

Wes: Well, I don’t want to write these in terms of theta and r. I can, but...but I don’t wanna. That’s an x. Um...

TA: Which coordinate is x?

7:30

Wes: (writing) x, y.

TA: Okay.

Wes: Tension in x... I mean...hold on. Tension times some angle.

TA: Can you define the angle in this picture?

Wes: Probably, would you like me to?

TA: Yes.

(Wes writes)

8:11

Derek: You have two forces called Ts?
Wes: I’m getting there. Um... this is Ts [inaudible] See, this is where I went last time. I didn’t like myself.

TA: Do you agree with this, is this what you would do?

Derek: I don’t know. Sure.

TA: What would you do?

8:45

Derek: I don’t know. I mean it’s as good a one as any. I don’t see why not.

TA: Okay.

Wes: (writing) That’s a sum, not an E.

TA: I just want to know that what’s going on here is indicative.

Derek: Sure, why not?

Wes: I don’t know what indicative means. I’ve heard it though. My brother would know, cause he’s smarter than I am.

TA: Indicates, what both people think.

Wes: Indicative, indicates. I guess so. (writing) Uh, which direction, up and down? Okay. Fe minus, remember this is positive.

TA: Right. Okay.

9:25

Wes: (writing) Umm...Ts, this is an s. I hope I’m drawing these with some sort of.

TA: That’s fine.

Wes: Ummmay. Now we have those.

(long pause to 10:13)
Wes: Sorry, I was just thinking.

TA: What were you thinking? Tell us.

Wes: Energy. But I'll ignore that, now.

TA: Okay. What do you think. Where would you go from here?

Derek: This is why I would have it in polar coordinates. Because then, we take (writing) sum of the forces equals ma, right? So, m d theta (writing)....I'm going to call this x....

TA: Can you say aloud what you're writing just so it gets...

Derek: Uh, yeah, just a second.... Alright, so yeah, just do simple, yeah, second law from right here, sum of the forces equal to ma. Sum of the forces in the x direction is equal to ma x. So, we just take m d theta squared in the x direction times m dt squared in...no, ah, wait, if we're doing this in polar, you don't need an x. So...equal to ma theta.

11:30

Wes: Sum of forces in theta.

Derek: Right.

Wes: Positive theta.

Derek: Then we just solve this equation. Which would be...

TA: Which direction on this picture, if you were in polar coordinates, what would your angle theta be?

Derek: Which one, this one?

TA: Yeah. In this picture.

Derek: That...
Wes: It’s technically, for clarification, we want two different angles.

Derek: Actually, it would be this angle, right here. Cause, yeah...opposite angles.

Wes: Opposite interior.

Derek: Yeah.

TA: Mmkay. So if we call that theta.

Derek: Yeah.

12:10

TA: What’s the theta direction?

[inaudible as Wes and Derek start speaking]

Wes: It gets confusing.

Derek: That’s zero degrees, and theta goes in this direction.

TA: So with this, so if I asked you do draw a, ah, a coordinate system and asked you to draw the unit vectors, what direction would the theta unit vector point?

Derek: That’s why I have to keep the x’s out. Okay. That’s fine.

TA: Uh.

Derek: Well. (pause) Theta has to point in this direction.

TA: Can you draw that in the diagram?

13:09

Derek: So theta at this point.

Wes: Whoa whoa, a unit vector for what?

TA: To describe the direction of the theta coordinate.
Derek: It moves...It moves parallel to the path...of the motion--of the object.

TA: Do you buy that?

Wes: Yeah, I buy it, but that's not...

Derek: No, by what she just said, we've done something wrong. So.

Wes: But that's--

Derek: No, that's wrong, it's wrong.

Wes: What?

Derek: No, it's wrong.

Wes: Will you shut up?

Derek: No, it's wrong.

Wes: Positive theta, yes is exactly where this object moves. But I was just curious. The unit vector for theta. It's like asking a unit vector for x in cartesian.

14:00

Derek: Unit vector for x moves in the x direction.

TA: Okay, so..

Wes: So, to actually write it out, or draw it? Cause if you drew it, that's, that's what it is.

TA: Okay. That's, that's what I was asking, what's direction is theta.

Wes: Can you draw this in a little more artistic format?

TA: (writing) So if the mass is over here... and grew..

Wes: That's ok.

TA: What direction would theta hat be in this case.
Wes: Depending on initial conditions, it’s either this way or it’s this way.

These are parallel.

(phone rings)

Wes: That’s my phone, ignore that.

TA: Okay.

Wes: Who is that? Oh, it’s some kid, I have to fix his computer. Anyway.

14:46

TA: So how do you know which direction theta hat is?

Wes: Now? Well, positive theta hat is that way.

Derek: So the unit vector would be in that direction.

TA: So what if the mass is heading back down toward equilibrium?

Derek: The unit vector is always positive, it just, but the value itself can be negative.

TA: Ok.

Derek: Multiplied by the unit vector.

TA: So it doesn’t matter if the mass is moving this way or this way?

Derek: Right.

TA: Theta hat is always this way?

Wes: Well in that case—well...no.

TA: Well?

Wes: Okay.
Derek: Just think of i hat, k hat and j hat. Those are always positive. In
the positive direction, the value or magnitude of it can be negative,
though.

Wes: Well, yeah, in that case, it’s either way.

TA: So you’re saying it’s in one direction, but you’re saying it’s either
way.

Wes: No he’s saying it’s either way and I’m saying I agree with him–

Derek: No

Wes: –even though I accidentally wrote it this way.

Derek: It’s always in one direction. The unit vector’s always in one
direction. The coefficients of it can be a negative. Which make
it point in the other direction. Make the actual vector point in a
different direction.

Wes: Well, i hat, just i hat, is one direction?

Derek: Yeah, have you ever seen it point in another direction?

Wes: (shrugs) Yeah. Anyways, we have this, this positive i hat, so you’re
saying i hat is just this way–

Derek: Yeah.

Wes: and if you want to make it negative, you put negative on i hat–

Derek: Right.

Wes: –and it goes this way.

Derek: Which is technically negative one times i hat.
Wes: Well, yeah. So why the hell would you say it doesn’t matter?

Derek: I didn’t say it doesn’t matter.

Wes: Yes, you did. Kay, forget it. End of discussion. We’re agreeing with each other. This is positive i hat. I mean, uh, theta hat. Yeah, ok.

TA: So, positive theta hat always is–

Wes: –that way.

TA: –this way. Always this way.

Wes: Yeah.

TA: Ok. (pause) So.

Derek: So, um.

Wes: We need gravity.

Derek: (pointing to his solution steps) No that’s wrong.

17:03

Wes: Why?

Derek: Because it doesn’t have all the terms in it.

Wes: I know that.

TA: Is there any tension in the theta hat direction?

Derek: No, tension’s always perpendicular.

Wes: Really? Wow...that would make sense. Anyway. Sum of the forces, which you took out your little theta...

TA: Which way is the r hat direction?

Wes: What’s r?

(pause)
Derek: What the hell you smoking, boy?

Wes: This r?

Derek: Yes.

Wes: r hat.

17:57

Derek: I’ll give you a hint, it’s in the same direction, where we pointed zero. Where it used to be.

Wes: I don’t want it to be.

Derek: It’s what you drew. It’ll be your y.

Wes: Isn’t it in all directions?

(pause)

Derek: No. (pause) It points downward. This way it makes gravity really easy to figure out.

Wes: That, so r hat should point here.

Derek: Right.

Wes: (writing) This is r hat. That’s a hat.

TA: So does the direction of r hat ever change?

Derek: No.

TA: Does the direction of theta hat ever change?

Derek: No.

Wes: Guess not.

TA: Right here you have theta hat pointing this way, and here you have theta hat pointing this way.

Wes: Doesn’t change.
Derek: In a polar coordinate–

343 TA: This way isn’t a different direction than this way?

Wes: The universe is spinning this way, so.

345 (pause)

19:09

347 Wes: So, yeah...I just... Yes, the direction’s changing.

TA: But not for r hat.

349 Wes: I guess not, yeah.

TA: You guys both agree on that?

351 Derek: Sure.

Wes: No.

353 TA: No?

Wes: No, I say that r hat is whatever direction it needs to be at that point.

TA: So you would say, which direction would you say r hat was?

357 Wes: It’s in all directions, radially outward, at the same time.

TA: Okay. And you’re saying r hat is just down.

359 Derek: Sure. This is why I like lectures. I don’t like tutorials.

Wes: You don’t like tutorials?

361 Derek: No. You’re guessing.

Wes: Yes, I’m making educated guesses. On what I know. If I get proved wrong, I’ve learned something.

Derek: Yeah, I’ve learned not to say anything.
Coordinates-1

1  Rose 0:00:00 (everyone is writing on their papers) But the equation they have is in terms of theta, so we don’t want to... 'cause those (points at something on board) are like broken up in terms of x and y
2  TA 0:00:18 How do x any y (unclear)
3  Jessica 0:00:19 (speaking over TA) what’s in terms..
4  Rose 0:00:20 Well I mean the way he... seperated those mg components
5  Jessica 0:00:27 That’s sine theta (points at something thing on board)
6  TA 0:00:28 yeah
7  Rose 0:00:29 Oh... but I mean... nevermind.
8  TA 0:00:32 So, if I... if I draw my ball like this. I’m going to draw another ball, I’m going to make it smaller, um (draws small pendulum on board with ball to right of equilibrium), like this. We’re just drawing this for purposes of a coordinate system. How does my coordinate system go?
9  Vivek 0:00:55 We can just choose what we want.
10  TA 0:00:55 Any coordinate system? Some coordinate systems are easier than others, which one should we pick?
11  Vivek 0:01:01 Pick it so that... you have the forces along the x’s or y (points at T and mg on large pendulum diagram)
12  Jessica 0:01:05 Well, if we have thetas we should be using polar coordi-
13  nates
14  Rose 0:01:09 yeah
15  TA 0:01:09 polar coordinates?
TA 0:01:15 So, in polar coordinates when unit vectors are theta-hat and r-hat, which way is theta-hat?

Rose 0:01:23 It's along the
Jessica 0:01:23 (speaking over other student) Tangent

Rose 0:01:25 yeah, that's what I was trying to say, it's along the motion, tangent to the motion of... of the...

Jessica 0:01:32 tangent to r
Martine 0:01:32 So velocity

Rose 0:01:34 no it's perpendicular to r. Did you say theta-hat or r-hat?

TA 0:01:37 I said theta-hat
Martine 0:01:39 so, yeah, velocity's perpendicular to... to T and tangent

TA 0:01:44 So...

Jessica 0:01:46 yeah perpendicular

TA 0:01:47 here are two arrows that are perpendicular (draws arrows on ball perpendicular to string). Which one gets to be theta-hat?

Rose 0:01:51 that one (JW points at inwards one)

TA 0:01:52 this one? (points at same vector, then erases other)

Vivek 0:01:53 yeah... I'm just still (unclear)

Martine 0:01:58 I don't know why either

Vivek 0:02:00 because... (unclear) equation, this has theta (indicates angle between string and equilibrium on small diagram)

Martine 0:02:03 he's writing on the board again (TA redraws vector she just erased)
Jessica 0:02:05 I don’t think... does it matter? I mean all it will be is a
negative or positive
Vivek 0:02:11 We’re taking this as theta, right? (indicates angle between
string and equilibrium on small diagram)
TA 0:02:14 Well, I don’t know.
Jessica 0:02:17 Won’t it be negative or positive? I mean...
Rose 0:02:20 Well, where do we want to call theta zero? If theta’s zero in
the center (places pencil to indicate vertical) don’t we want theta-
hat to be that way (outwards pointing vector)... so that its... I guess
I don’t understand what the difference would be.
TA 0:02:30 They can’t both be theta-hat.
Vivek 0:02:32 yeah
TA 0:02:34 ’cause that would say that theta-hat has two different direc-
tions.
Rose 0:02:40 so... but its arbitrary though, isn’t it?
TA 0:02:42 Pretty much, yeah. So you have to decide, and I’m going
to suggest that as a table we decide... unified, so that everybody
agrees
Jessica 0:02:52 Well, I think in this case, since it’s way on the end, it’s
going to be coming this way (gestures towards vertical) so we should
have it, on closer to this line
Rose 0:02:57 (TA points at inward pointing vector) yeah.. pointing back
to equilibrium
TA 0:03:05 okay (erases outward pointing vector)
Vivek 0:03:05 if it’s (unclear) I will buy that

TA 0:03:07 What about r-hat?

Jessica 0:03:09 It’s along that, it’s along the string. (JW indicates the
direction along the string)

TA 0:03:12 This way? (draws vector along string pointing towards pivot)

Jessica 0:03:13 (affirmative hum)... No, the other way

TA 0:03:16 This way? (draws vector along string pointing outwards,
erases inward vector)

Jessica 0:03:16 yeah

TA 0:03:22 and where is (0,0)?

Vivek 0:03:25 up there (points at pivot)

TA 0:03:27 Up here? (points at pivot)

Vivek 0:03:28 (unclear)

Rose 0:03:31 Where is what 0?

TA 0:03:32 (0,0)

Rose 0:03:33 Oh. Where is the origin? (Vivek points at pivot) Yeah.

TA 0:03:37 Up here? (points at pivot)

Rose 0:03:38 Well, it would make more sense to do it where the ball is
(TA draws + at pivot), though, wouldn’t it? So like down and in
the center?

Martine 0:03:45 Yeah, down there (Megan points at ball’s equilibrium
position)

TA 0:03:48 Down there? The ball’s not right here (at equilibrium)
though, its right here.
Rose 0:03:49 Well, but I mean, when the ball is at equilibrium. Would it make sense to put that (0,0)? In the center?

Jessica 0:03:57 yeah

Rose 0:03:57 Although then r would be...

Jessica 0:03:59 Not there (TA draws + at equilibrium), that’s not where it will be. Found it (unclear) (points at center line on current ball level, TA draws a + there)

Vivek 0:04:03 But how about the r-hat?

Jessica 0:04:06 ’cause that means it’s making the string longer and that (unclear)

Rose 0:04:10 Well you could go like that so it’s not going to end up level (uses pencil to indicate motion of string)

Martine 0:04:14 yeah, it should go...

Rose 0:04:16 it’s going to be like an arc

Jessica 0:04:18 Oh, okay. (TA erases + above equilibrium)

Vivek 0:04:22 (unclear) I could do this one (points at pivot), because from here you measure r (indicates length of string)

Rose 0:04:27 Oh yeah. That makes sense. Yeah.

Vivek 0:04:33 Because they’re going to be (unclear) (indicates string) because there (0,0) is up at the center

TA 0:04:40 Right side of the table? I see nodding on the left side of the table.

Martine 0:04:43 Sure.

Ed 0:04:45 Why is r away?
Vivek 0:04:48 Because... in coordinates (unclear) system, which will be
carte... polar coordinates? (unclear)

Martine 0:05:00 Does it matter? a lot, like...

TA 0:05:02 Not really, no. (general laughter) Um, in polar coordinates
the radius, r, of the vector is always measured outward from the
center. That’s just convention.

Ed 0:05:15 Okay.

TA 0:05:16 So if you decide to make this your center, then outwards from
here is in that direction. (points outwards along string)

Martine 0:05:23 So that would be i, ah... r-hat... okay

Rose 0:05:30 So we said the origin is... right there (points at pivot), okay.

(TA erases + at equilibrium)

Ed 0:05:38 So many things to label on one small graph

TA 0:05:41 So...

Vivek 0:05:43 is that what you had (unclear) points at theta-hat vector

TA 0:05:51 Where is theta-hat... or where is...

Jessica 0:05:53 Its ah... arbitrary. That’s what we said.

TA 0:06:00 It does... Honestly it does not matter.

Rose 0:06:02 The direction?

TA 0:06:03 As long as you’re consistent... and as long as its perpendicular
to r, then it doesn’t matter. But, its good if you’re all consistent
about it.

Rose 0:06:11 So we have it going in towards the center.

Martine 0:06:13 Yeah.
TA 0:06:15 If the ball were over here (draws ball to left of equilibrium)... what direction would r-hat be? Everybody point. (everyone points outward along string at new location)

Martine 0:06:23 r-hat? or theta-hat?
TA 0:06:25 r-hat. That way? (draws vector along string pointing outwards)

Martine 0:06:27 Yeah.

TA 0:06:30 What direction would theta-hat be?
Martine 0:06:34 Coming back.

Ed 0:06:36 that way (indicates a vector pointing towards equilibrium perpendicular to string)

TA 0:06:37 This way? (draws indicated vector)
Vivek 0:06:38 yeah

Martine 0:06:38 yeah

TA 0:06:39 So theta hat changes direction?

Rose 0:06:42 It should still be in the same direction, shouldn’t it?
Vivek 0:06:45 Because its (unclear) (indicates angle between equilibrium and right position of ball)

Rose 0:06:49 Yeah, but theta-hat’s just the unit vector (TA draws vector in opposite direction to one already drawn)

Jessica 0:06:51 We said that it didn’t really matter
Rose 0:06:53 So wouldn’t you want it to point this way (points at vector pointing away from equilibrium) so that over here (points at right ball position), like theta’s zero in the (indicates equilibrium position), so over here theta would be positive (right of equilibrium) and over here it would be negative (left of equilibrium). So don’t we want it to point that way? (away from equilibrium)

Vivek 0:07:02 Increase this is to that, then, then this will raise theta (marks angle between strings and equilibrium as theta on both sides of equilibrium)

Rose 0:07:05 Yeah, so we want the unit vector to point this way (away from equilibrium) so that theta over here (left of equilibrium) is minus.

Martine 0:07:10 It’s still the same though

Vivek 0:07:10 (speaking over previous student) what about negative would that cause the equation to change (points at angle left of equilibrium)

Rose 0:07:14 Well if the direction is in the same direction as theta, then theta will be positive though, wouldn’t it?

Martine 0:07:21 No?

Vivek 0:07:22 If you take the theta down from this there’s (unclear) then um... If you take the theta from this vector then (indicates equilibrium line), then it’s... because you could take theta at here to be zero (points at equilibrium position). Ah, take this direction (right of equilibrium), then it looks like... this way. (other students begin speaking over) And uh...
Rose 0:07:38 I disagree.

Martine 0:07:38 Should this have a point though.

Martine 0:07:39 Yeah, she does have a point.

Ed 0:07:46 We’re starting from second law. Right?

TA 0:07:49 Right now we’re starting from... previous to second law, what coordinate system are we using.

Ed 0:07:56 Oh, yeah, you got to pick a coordinate system.

Rose 0:07:58 And plus... I don’t know if that has anything to do with it, but don’t you want r and... r-hat and theta-hat to have the same orientation no matter where you are?

TA 0:08:05 What do you mean?

Rose 0:08:06 Well, like, over here (right of equilibrium) if you cross them, ’cause they like kind of determine you plane hand

TA 0:08:12 Okay

Rose 0:08:13 If this is pointing over here (vector pointing towards equilibrium to the left of equilibrium) then you plane flips over

TA 0:08:15 What do you mean by “plane flips over”?

Rose 0:08:17 Well, over here your cross product is (uses right hand rule)... pointing in, I think, over here (uses right hand rule) its pointing out.

TA 0:08:29 You want, you want your coordinate system to have the same handedness no matter where you are?

Martine 0:08:33 No.
Rose 0:08:39 Hang on...

TA 0:08:40 But, that’s what you said.

Rose 0:08:41 Yeah, it just seems like

Martine 0:08:41 Hold on. I don’t think so, when you have this working with circuits you don’t

Vivek 0:08:45 The only reason I did was, I see this one as negative theta (angle to left of equilibrium) so I thought that would change the direction. I don’t know what that is.

Rose 0:08:57 Okay.

Vivek 0:09:00 You need to say something. We are stuck.

TA 0:09:03 What if...

Vivek 0:09:05 what if, what if, I knew you couldn’t

TA 0:09:07 What if...

Ed 0:09:08 I take some simple algebra...

TA 0:09:11 I... um... I drew... our favorite, the x-y plane. (draws x-y plane)

Vivek 0:09:19 yeah

Ed 0:09:21 Which would just we just learned last semester.

TA 0:09:24 And I... (places a point in 4th quadrant)

Jessica 0:09:24 Really?

Ed 0:09:25 No.

Unknown 0:09:26 (laughter from whole group)
TA 0:09:30 And I put my, and I want to know what direction is x-dot if I’m right here (points at point in 4th quadrant), er... x-hat. Everybody point. What direction is x-hat if I’m right here? (points at point in 4th quadrant)

Jessica 0:09:46 It could be either way.

TA 0:09:47 Oh, let me label it here. (puts arrows on axes)

Rose 0:09:52 It’s in the direction of positive x.

TA 0:09:55 In the direction of positive x. x-hat is this way? (draws vector at point in 4th quadrant pointing in positive x direction)

Rose 0:09:58 Yes.

TA 0:10:00 What about if I’m over here? (draws point in 3rd quadrant)

Rose 0:10:01 Still this way.

TA 0:10:03 Still that way? (draws vector at point in 3rd quadrant parallel to one in 4th)

Martine 0:10:04 Yeah... yes. (TA draws vector at point in opposite direction)

Rose 0:10:05 x-hat is the same everywhere.

Martine 0:10:07 Yeah, x-hat...

Rose 0:10:09 the x-hat direction doesn’t change

Martine 0:10:11 yeah, it’ll be the same.

Vivek 0:10:11 so then (unclear) because it’s negative there (indicates a negative arrow on x-axis)

Martine 0:10:16 (speaking over previous student) Oh, it’s just symetrical, but the...
Rose 0:10:19 If x-hat was pointing this way (negative direction), then you’d... Okay, find your value of x at this point (in 3rd quadrant). You’d have this magnitude times this direction, which would make it... positive, but you want it to be negative so it has to point this way (in positive direction).

Martine 0:10:35 yeah, I think it’s the same way.

Rose 0:10:40 I’m glad someone agrees with me. (group laughter and unclear comments over top) What do you think?

Ed 0:10:45 I have no idea.

Rose 0:10:46 You’ve been too quiet.

Ed 0:10:47 I’m confused. Could you draw another arrows vector

Vivek 0:10:54 hehe... somebody support me, okay. I give up.

Rose 0:10:59 Well, when you use vectors...

Jessica 0:10:59 Why do we want it negative?

Rose 0:11:00 When you use i-hat, j-hat, and k-hat, they always have the same direction. You have your i, j, k when you do your whatever, your cross products and stuff. Those don’t change.

Vivek 0:11:09 okay

Martine 0:11:11 and this one changes because you pass through the y

Rose 0:11:13 right. cause that would make everything positive everywhere if you change the direction

Vivek 0:11:19 okay. Okay, I’ll buy that. I think.

Ed 0:11:23 So in a basic... in a basic explanation why is that the same direction? (points at positive vector at point in 3rd quadrant)
Vivek 0:11:27 If its constant (unclear)

Rose 0:11:28 Its just the way it’s defined. (other voices speaking over) If you need a vector

Ed 0:11:31 Once you define the direction you...

Rose 0:11:34 Well, if you’re to the left of the x-axis you want a negative x (points at positive vector at point in 3rd quadrant)

Jessica 0:11:37 Oh, okay.

Martine 0:11:39 You could say that 0 is (unclear due to next speaker)

Vivek 0:11:43 (unclear, speaking over previous speaker) (TA erases negative vector at point in 3rd quadrant) wouldn’t the magnitude be negative then?

Rose 0:11:46 Magnitude is always positive. (unclear) squared.

Jessica 0:11:50 You square it, you can’t...

TA 0:11:55 (labels vector in 3rd quadrant as x-hat) Okay, so... its true that this is x-hat and x-hat. One way to think about it is x-hat, always that way. (gesture off camera)

Rose 0:12:10 So it’s the same for theta-hat, right?

Martine 0:12:14 So it won’t change.

TA 0:12:15 Yes, but...

Ed 0:12:17 There’s always a ”but”

TA 0:12:18 Yes but... Can I erase my x-hat stuff.

Rose 0:12:22 Yeah, I got the x-hats.
TA 0:12:28 (erases x-y plane) Okay, so we know that this \( \mathbf{r} \)-hat (right of equilibrium) sort of points off that way (along string) and this \( \mathbf{r} \)-hat (left of equilibrium) points off that way (along string). You guys didn’t disagree about the \( \mathbf{r} \)-hats. Is it okay that they’re pointing off in different directions?

Vivek 0:12:43 They’re always out of center.

TA 0:12:46 Ah! Always out of the center?

Unknown 0:12:48 (affirmative sounds from whole group)

TA 0:12:49 Okay. What about the orientation of \( \mathbf{r} \)-hat and \( \theta \)-hat?

Ed 0:12:53 Well, they always have to be perpendicular.

TA 0:12:54 They always have to be perpendicular, but we have two choices for perpendicular.

Martine 0:12:59 That’s what we’ve been arguing for the past half-hour.

Ed 0:13:03 This kind of perpendicular (away from equilibrium) or that kind (towards equilibrium).

TA 0:13:05 That’s true. There’s a way to decide.

Martine 0:13:09 What’s the way?

Vivek 0:13:11 Okay, if that was true then it would be that around (erases vector pointing towards equilibrium on left side of equilibrium)

TA 0:13:20 Wait a sec...

Rose 0:13:23 I think its duh...

Ed 0:13:24 (speaking over previous student) I think stick this way. Like that. (towards equilibrium on right of equilibrium)

TA 0:13:27 Like this? (towards equilibrium on right of equilibrium)
Rose 0:13:28 And it just stays that way at the end of the... so the coordinate system is fixed to the ball.

Ed 0:13:34 Yeah

Martine 0:13:34 Yeah

Rose 0:13:36 And it just stays there

Ed 0:13:37 Yeah!

TA 0:13:38 So what if the ball were (unclear) (gestures indicate a ball swinging left to right)

Rose 0:13:39 It's like "Yes!"

TA 0:13:41 If the ball is moving towards me...

Martine 0:13:43 The ball’s coming back now.

TA 0:13:45 Like if the ball is moving back towards me. In theta, is it going in plus or in minus?

Jessica 0:13:52 Minus. Minus?

TA 0:13:53 Minus?

Ed 0:13:55 Yeah, cause it’s going in the direction opposite the coordinate thingie.

TA 0:13:59 Okay.

Ed 0:14:01 I know "thingie" is not a technical term

Rose 0:14:04 We know what you meant though.

Martine 0:14:05 Okay, then if the ball was coming back then theta-hat would be the opposite direction? Is that what we just said?

Rose 0:14:10 No.
Martine 0:14:12 Okay, so theta-hat would be this direction for ever, it wouldn’t matter if its coming back or going forward. Okay.

TA 0:14:20 So we just said that theta is measured around this way. (draws clockwise wrapping arrow off to side)

Martine 0:14:27 Okay.

TA 0:14:28 And r is measured… (draws arrows pointing out from center of wrapping arrow)

Ed 0:14:30 Like that. Radially outward.

TA 0:14:32 Yeah, outward. Okay, where is theta equals zero?

Ed 0:14:39 Right... there. (points at pivot)

Rose 0:14:42 In the center?

TA 0:14:43 (affirmative hum)

Rose 0:14:44 Can’t we just define it to be zero where ever we want it?

TA 0:14:46 We can define it where ever we want but we have to pick.

Ed 0:14:49 Okay (unclear)

Rose 0:14:49 (unclear) the center.

TA 0:14:50 The middle?

Rose 0:14:50 Yes.

Ed 0:14:51 Right in the center (TA draws center line and labels it as theta = 0). So therefore only in the... counterclockwise direction is positive?

TA 0:15:10 That looks like clockwise to me.

Vivek 0:15:13 It’s clockwise.

Rose 0:15:14 Well, yeah, it’s... That’s positive. (swings hand clockwise)
Ed 0:15:17 Oh. Well, I was thinking that line (points at something on his paper), well you see its going that way

Rose 0:15:21 Oh. You’re sign is going the other way from that one because we’re looking at it upside down.

Ed 0:15:25 Yeah. (laughter from group) Okay.

TA 0:15:30 Does every one have a unified picture of the coordinate system?

Rose 0:15:31 Whoops... my r is going backwards. r is going out.

TA 0:15:50 I’m going to copy it over here. (draws small diagram with coordinate information in corner of board)

Martine 0:15:51 Is it easier than just working with the... the x and y coordinate? I mean I know we can’t because it’s asking for this equation, but it seems so much more...

Rose 0:16:02 Probably if we were more used to polar coordinates, it’s just that we never use it so...

TA 0:16:18 Can I erase this? (affirmative sounds from the group) (TA erases small working diagram of pendulum) I made a little picture here.

Ed 0:16:22 Okay, I’ll copy it down.

TA 0:16:25 We can also put it on the (unclear)

Ed 0:16:27 Well, cause see, this is... I don’t want it, I don’t want to do that. I want to keep the drawing clear.

Vivek 0:16:36 We’re doing (unclear) let’s do it fast, because its (unclear, but context indicates he’s stating the time)
TA 0:16:43 We just spend 20 minutes trying to figure out a coordinate
system.

Ed 0:16:45 That’s cool.

TA 0:16:46 Alright, now that we have a coordinate system what can we
do?

R-forces

Ed 0:00:00 Uh... (some writing on the worksheet, pencil tapping)

Vivek 0:00:23 (looks something up in text book)

TA 0:00:24 Okay, where are we? Oh right making into r-hat and theta-
hat

Ed 0:00:28 Yeah the forces or whatever

TA 0:00:30 (affirmative hum)

Ed 0:00:31 Oh so we should have like T r-hat and mg cosine theta-hat,
r-hat, theta-hat

TA 0:00:42 (laughter) it’s true it’s a binary choice. You have to pick one

Ed 0:00:44 Yea but, like, um... at... this (pointing at something on work-
sheet) is in the negative r-hat direction so that would be negative

unless you (unclear) it its along it

TA 0:00:53 yup

Ed 0:00:54 so that’s part of

TA 0:00:56 that was the r-hat

Ed 0:00:58 the r-hat portion of the force summation or something
TA 0:01:01 yeah, the kind of place where you do the sum of forces in x

    and then the sum of forces in y

Ed 0:01:06 right

TA 0:01:06 now you’re doing the sum of forces in r and this is the sum
    of forces in theta

Ed 0:01:10 so you have minus T r-hat plus mg cosine theta r-hat (writes
    on worksheet)

TA 0:01:20 what do you get for theta-hat

Jessica 0:01:23 what is this equal to? Force?

TA 0:01:25 that’s for the sum of forces in the r-hat direction, um... so...

Jessica 0:01:30 okay I think I did some thing different

TA 0:01:34 what do you have, here marker

Jessica 0:01:38 (writes (T sin theta) theta-hat + (T cos theta - mg)R-hat
    on board)

TA 0:02:00 It appears that you have made your r-hat always in the down
    direction

Jessica 0:02:07 no.. I made my r-hat

TA 0:02:11 or always in the up direction

Jessica 0:02:20 have I?

TA 0:02:21 yeah

Jessica 0:02:22 well that’s not a squared (unclear)
TA 0:02:26 that’s not the same as everyone else has... see by having minus mg r-hat (points to -mg on board) you’re saying that mg is always in the minus r-hat direction meaning that down is always in the minus r-hat direction meaning

Ed 0:02:44 the r-hat would have to be up, yeah

Jessica 0:02:47 oh, so it should be this way then (changes signs on r-hat components written on board)

TA 0:02:50 now your r-hat is always down

Jessica 0:02:53 well, that’s what it is

TA 0:02:55 it looks like its sort of down at an angle over here (points to diagram of pendulum to right of equilibrium)

Ed 0:03:00 it’s down here (points to diagram of pendulum at equilibrium)

Jessica 0:03:05 oh... so you’re saying this would (unclear) (erases +mg)

TA 0:03:13 sort of... what did you guys get?

Ed 0:03:19 well, I haven’t done the theta-hat

TA 0:03:21 okay, what did you get for your r-hat

Ed 0:03:23 I have a minus T 1 and an mg cosine theta 1

TA 0:03:28 you have what?

Ed 0:03:30 minus T r-hat and cosine theta r-hat.

Vivek 0:03:34 How did the other...

Ed 0:03:36 It’s minus because the tension is in the opposite direction of r-hat (points to T vector on diagram)

Vivek 0:03:43 that’s the minus T-hat er.. T r-hat
Jessica 0:03:46 okay minus, but why do you get... how do you get mg for theta?

Ed 0:03:50 well I’ve got mg cosine theta ’cause mg cosine theta is in the direction of r-hat... it’s along r-hat... and it’s positive because it’s pointing in the direction that r-hat’s pointing

Jessica 0:04:16 (very softly)I don’t see the cosine theta

TA 0:04:19 Okay if I looked at...

Jessica 0:04:22 My drawings could be bad, but I don’t see it

TA 0:04:23 If we look at this ball right here (points to diagram of pendulum to right of equilibrium) right, this way is r-hat (draws r-hat towards pivot) and this way is theta-hat (draws theta-hat in clockwise sense) and this way is mg (draws mg vector)... oops, I drew my r-hat backwards (erases r-hat and redraws it in opposite direction), that’s better, this way is r-hat and this way is tension (draws T vector)

Jessica 0:05:01 So I have my free body wr...? I have it right, right? My free body diagram?

TA 0:05:07 Umm... yeah but you need to put your r-hat and theta-hat on your diagram so that we know how they relate to it.

Jessica 0:05:19 Are you allowed to do that?

TA 0:05:21 Are you allowed... You’re always allowed to put a coordinate system on your free body diagram.

Jessica 0:05:24 okay

Ed 0:05:25 yeah, I think it was required when I took physics under [a physics professor, not the one for this course]
Vivek 0:05:29 hey... the x and y... so how did I get to missing ends up
(unclear)

TA 0:05:44 (questioning hum)

Vivek 0:05:45 it’s missing a big T... sigma is (unclear)

TA 0:05:55 so it’s sum of forces equals ma?

Ed 0:06:02 I haven’t gotten the theta-hat part yet

TA 0:06:08 let’s work through the r-hat forces. So what do I have for the
sum of forces in r? (writes F sub r = and then writes down forces
as the students say them)

Vivek 0:06:13 minus T... plus mg cosine theta... do we need to add
r-hat?

TA 0:06:29 Um...

Ed 0:06:30 Well, you already denoted the...

TA 0:06:32 Yeah, I already called it sub r. I could... (adds r-hat to both
sides of equation)

Ed 0:06:35 have done that...

TA 0:06:46 Okay, sum of forces equals ma (adds = ma to equation of
forces), but since this is acceleration in the r-hat direction, I’m going
to call it a sub r (adds subscript r to a)

Vivek 0:07:01 oh...

TA 0:07:03 (unclear)

Ed 0:07:08 yeah

TA 0:07:09Alright, let’s keep going, let’s try the theta-hat ones

Ed 0:07:09 Oh good that’s (clip ends)
Appendix C

TRADITIONS AND THEORIES

In physics, a theory is a large-scale construct that defines, describes, and predicts a wide range of physical behaviors. A model is a smaller-scale construct with many of the properties of a theory, but a significantly smaller scope. For the few behaviors that a model describes, it describes them more simply than the corresponding theory may describe them, but it may also lack some richness of detail irrelevant to the model that the theory allows. For example, a complete description of friction between a block and a ramp might include measurement of fine-scale texture of both block and ramp and calculations of the inter-molecular forces based on electrostatic attraction and repulsion. In contrast, a simple model of kinetic friction reduces this complexity to a single parameter, μ_k, a proportionality constant which relates the force of friction, F, to the normal force on an object, N. This constant can be experimentally determined for the ramp and block. This model sufficiently describes the force of friction on a block. The electrostatic theory is also applicable in principle, yet is much more complex. However, it is also applicable to a wider range of problems.

Just as physics makes the distinction between small-scale models and large-scale theories, cognitive science makes a distinction between small-scale theories and large-scale theoretical frameworks. Kuhn[136] would term the smaller-scale theories and the larger paradigms, noting that a given field can only support one paradigm at a time, and that the governing paradigm controls which questions are valid to investigate in addition to which modes of investigation and proof are relevant. Laudan [101] generalizes Kuhn’s paradigms into research traditions, noting that a given field can and ought support more than one at a time. Laudan’s
distinction (built on Lakatos’[137] earlier research programmes ) is important to multi-disciplinary fields such as PER, whose constituent fields may have different goals and methodologies.

Using Laudan’s language, Resource Theory is a research tradition made up of several interrelated theories which integrates reasoning and knowledge organization and activation to account for various users’ different responses at different times to different, possibly internal, stimuli. For this paper, I will use Laudan’s language of theory and tradition. To aid in the distinction, Traditions are capitalized and theories are not. Resource Theory is so called because it comes from physicists, who use the physics language of models and theories, not Laudan’s language. I don’t use the physics terminology because I wish to reserve models for the application of theories to data: modeling a student using plasticity theory, for example. Furthermore, as the material in this dissertation draws from and relates to several fields, just as PER does, I find the more general language more appropriate.

This dissertation draws primarily from two Traditions, Pieces Theory and Process/Object Theory. To better discuss the expansions to Pieces Theory that it offers, some further terminology about the strengths, limitations, and methods of comparison between theories and Traditions is necessary.

Problem-solving effectiveness

A full description of a Tradition includes the theoretical details, experimental methods, ideas of sufficiency of solutions, and definitions of relevant and irrelevant behaviors. The “domain” of a Tradition can be phrased as the list of questions that it should solve. Questions that it has already solved are termed “solutions”. Questions that no Tradition has solved are “problems”, and questions that other Traditions
have solved, but this particular Tradition has not, are termed “anomalies”. One marker of a mature Tradition is a paucity of anomalies and a plethora of solutions.

Using this language, theoreticians expand the number of problems by phrasing new questions (possibly in response to new data generated by experimentalists) while they reduce reduce the number of anomalies (either by shrinking the domain of a Tradition or by converting them to solutions). Converting anomalies to solutions is generally preferable to shrinking the domain of a Tradition because scientists generally prefer more general theories. Similarly, experimentalists generate research methods and new data and analyses to aid the theoretician’s work. Together, they increase the problem-solving effectiveness (“PSE”) of the Tradition.

A mature Tradition thus has high PSE, but because of the paucity of anomalies, it has few theoreticians working on it and therefore has a small change in PSE with time. An experimentalist might choose to use this Tradition because it seems “safe” or conventional. A newer Tradition has few solutions and may have many theoreticians working to increase them: it thus has low PSE and high dPSE/dt.

PSE is always measured relative to other Traditions in the same domain. As one Tradition defines a new problem, then all Traditions in that domain have a new problem. As one Tradition solves that problem, all other Traditions with that problem now have a new anomaly. Traditions fall out of favor through the weight of intractable anomalies or because their domain has been whittled away to insignificance.

Limitations

There are two kinds of limitations on theory: within-Tradition limitations, and without-Tradition limitations. If a Tradition has a set of solutions and anomalies, then one of its constituent theories may solve, fail to solve (but purport to ad-
dress), or simply not address any specific solved question. Failures of this sort are termed “within-Tradition” limitations. If, on the other hand, one of the constituent theories fails to solve an anomaly (but purports to address said anomaly), these failures are termed “without-Tradition”. Clearly, within-Tradition limitations help theories align themselves within the Tradition but do not impact the PSE of the Tradition as a whole. Without-Tradition limitations are more grave, as they impact the Tradition’s PSE.

Comparing Theories and Traditions

Two theories may be compared if they describe or predict similar behavior. Two comparable theories may disagree in their descriptions or predictions. If they disagree on predictions, then a Popperian crucial experiment would strike down the offending theory[138]. If they disagree on descriptions, then scientists generally prefer the more parsimonious and complete theory. This simple limitation, while obvious, helps to narrow the field of all possible theories to ones which directly relate to the question at hand. As one example, Newtonian kinematics and special relativity are both theories of motion. In ordinary life, they both describe motion equally well, so Newtonian kinematics is preferred for simplicity. For high-speed or extremely precise calculations, Newtonian kinematics is insufficient, and relativity is a better theory.

Two comparable theories may also agree in their descriptions or predictions. In that case, evidence for or against one theory may be used as evidence for or against its compatriots to the extent in which they agree. In ordinary life, the equations for relativity reduce to the equations for Newtonian kinematics. Therefore, evidence for Newton’s theories also supports relativity in that regime.
Two theories can be indirectly compared if the implications of one theory affect the interpretation of the other, even if the theories proper do not directly concern the same phenomena. In an indirect comparison, one theory may extend or limit the province of another without directly affecting it.

With these two kinds of comparisons in mind, it is possible to compare Traditions in PER, MER, and cognitive science.
### Appendix D

**RESOURCES NAMED**

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explicitness, 64
flipping, 25
forces, 25, 26
force[sign, 3, 26, 32, 43–45, 47–49, 54, 55, 59, 96, 103, 114
function, 32
handedness, 95
knowledge-as-invented-stuff, 48
knowledge-from-authority, 24, 48, 87, 103, 105, 113
looking, 24
medical terminology, 41
money, 25
more plastic, 34
more solid, 34
natural, 59, 64, 65, 68, 70, 72, 74, 75, 77–79, 83, 85, 87, 90, 92, 98, 104, 112, 113, 119
numberline, 58, 70
object, 25
objects, 24
origin, 71, 78, 79, 91, 92
origins, 113
orthogonality, 58, 64, 65
part-for-whole, 25
past perfect progressive, 31
polar, 3, 64–66, 68, 70–72, 77–81, 83–85, 87, 91, 98, 103–107, 109, 111–113, 119
princess, 24, 26
properties, 56, 103
round, 24
separable, 116
social agree, 64
social agreement, 58, 64, 65, 75, 88, 90, 92, 93, 96, 142
span, 58, 64, 65, 68, 70, 72, 77–79, 83, 104, 113
spinning, 25
storytime, 24
use, 56, 103
value, 57, 58, 64, 65, 78–80, 83, 85, 90, 91, 93–96, 98, 104, 105, 109, 113
variable, 32
velocity, 25
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